State-Variable-Based Transient Circuit Simulation Using Wavelets

Carlos E. Christoffersen, *Member, IEEE* and Michael B. Steer, *Fellow, IEEE*

Abstract—**For the first time a state variable transient analysis using wavelets is developed and implemented in a circuit simulator. The formulation is particularly well suited to modeling RF and microwave circuits and is validated by considering a nonlinear transmission line. However, results indicate that still more research is needed to make this method efficient for the simulation of large circuits.**

I. Introduction

MULTIRESOLUTION analysis has generated considerable excitement in the engineering community because of its potential to efficiently model large systems with an overall coarse response but fine behavior in some regions. Wavelet basis functions are ideally suited to expanding such a response as higher order and more localized basis functions can be concentrated on the regions where the response varies rapidly. Multiresolution analysis has been used with a wide variety of modeling problems including signal processing and electromagnetics. It is important to know where wavelet analysis is applicable as this guides future development. This letter presents, for the first time, wavelet-based transient analysis incorporated in a general purpose circuit simulator. In circuits, voltage and current changes vary with time and location (*e.g.* node index) and so they can be modeled with few state variables by using variable resolution. In contrast, in conventional transient simulation the same fine time step is used at every node.

The use of wavelets for the transient analysis of circuits has been limited to the calculation of convolution operation in the transient analysis of simple circuits [1, 2] and the analysis of linear time-variant electrical networks [3]. Zhou *et al.* presented the spline pseudo-wavelet collocation method for simple networks in [4] and [5] (see also [6–9]). In the present study, we achieved wavelet transient analysis for arbitrary circuits by combining this collocation method with the state variable concept [10] in an object-oriented circuit simulator (*Transim*, [11]).

II. Background

This work uses the spline pseudo-wavelet collocation method described in detail in Reference [8]. In this method the unknown function is expanded in a wavelet series. The coefficients of the series are determined so the expansion fits the circuit response at a number of collocation points [6].

Consider the following system of nonlinear differential equations:

$$\begin{cases} \frac{d\mathbf{x}}{dt} = \mathbf{f}(t, \mathbf{x}) \\ \mathbf{x}(0) = \mathbf{x}_{t0} \end{cases} \quad (1)$$

where $\mathbf{x}(t)$ is an unknown vector function defined in a finite interval $I = [0, L]$, $L > 4$ and $\mathbf{f}(t, \mathbf{x})$ is a given nonlinear function. A scaling operation is applied to I to cover any interval.

Let $\hat{\mathbf{x}}_J$ be the vector with the wavelet coefficients of $\mathbf{x}(t)$:

$$\begin{aligned} \hat{\mathbf{x}}_J \quad = \quad & (\hat{x}_{-1,-3}, \ldots, \hat{x}_{-1,L-1}, \\ & \hat{x}_{0,-1}, \ldots, \hat{x}_{0,n_0-2}, \\ & \hat{x}_{J-1,-1}, \ldots, \hat{x}_{J-1,n_{J-1}-2}) \end{aligned}$$

and will be determined by satisfying interpolating conditions at interior knots called collocation points. Each component $x_i(t)$ of $\mathbf{x}(t)$ in (1) is replaced by its wavelet expansion form at each collocation point. Therefore we obtain the following nonlinear algebraic system

$$A\hat{\mathbf{x}}_J = \hat{\mathbf{f}}(\hat{\mathbf{x}}_J) \quad (2)$$

where A is a constant matrix that performs the derivative operation and $\hat{\mathbf{f}}()$ is a nonlinear vector function. Zhou *et al.* [7] use a relaxation method to solve the nonlinear equations.

This method then provides a *uniform error distribution* on the interval. By this property, large time steps can be used without introducing significant phase shifting between the approximate and exact solutions. The minimization of the error over an interval is the significant departure from conventional transient analysis where error is minimized at one time point at a time.

III. Concepts Behind Formulation

In wavelet-based transient analysis the unknowns are no longer voltages and currents but are now the coefficients of the wavelet expansion. Thus, a formulation of a reduced error function using a state variable approach can be derived as described below. One advantage of doing this is that the size of the resulting algebraic system of nonlinear equations is considerably smaller than that of (2). As well, good convergence properties can be obtained by defining all nonlinear functions as smooth functions [12].

Formulation of the problem is in two parts: a) the combination of the linear subcircuit with sources and the expression of the result in terms of wavelets; and b) the expression of the error function in terms of a wavelet expansion.

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The authors are with the Electronics Research Laboratory, Department of Electrical and Computer Engineering, North Carolina State University, Raleigh, NC 27695-7914, U.S.A., (e-mail: c.christoffersen@ieee.org and m.b.steer@ieee.org).

A. Source Formulation

The modified nodal admittance matrix (MNAM) of the linear subcircuit is now formulated in the time domain and integrated with the sources. Two square matrices of dimension n ($\mathbf{M}$ and $\mathbf{M}'$) are obtained, one for the unknowns and the other for its derivatives

$$\mathbf{M}\mathbf{u} + \mathbf{M}'\dot{\mathbf{u}} = \mathbf{s}. \quad (3)$$

Here $\mathbf{u}$ is the vector of nodal voltages and selected currents, $\dot{\mathbf{u}}$ is the corresponding derivative and $\mathbf{s}$ is the source vector composed of a 'fixed' time variant component $\mathbf{s}_f$ and a variable nonlinear component $\mathbf{s}_v$ that depends on the state variables:

$$\mathbf{s} = \mathbf{s}_f + \mathbf{s}_v. \quad (4)$$

Wavelets are introduced by considering the function $g(t)$ defined in I. The following square matrices $\mathbf{W}_J$ and $\mathbf{W}'_J$ can be defined:

$$\mathbf{g} = \mathbf{W}_J\hat{\mathbf{g}}_J, \quad \dot{\mathbf{g}} = \mathbf{W}'_J\hat{\mathbf{g}}_J \quad (5)$$

where $\mathbf{g}$, $\dot{\mathbf{g}}$ are vectors whose elements are the function and derivative values, respectively, at the collocation points and $\hat{\mathbf{g}}_J$ is the vector of the corresponding coefficients. J is the maximum subspace level being considered.

$\mathbf{M}$ and $\mathbf{M}'$ are now expanded using $\mathbf{W}_J$ and $\mathbf{W}'_J$, respectively, reduced by removing the redundant first row. Additional equations are obtained by noting that the product of the coefficients of each electrical variable times the first row of $\mathbf{W}_J$ is equal to the initial condition for that variable. The extra rows from these equations plus the sum of the expanded matrices result in a sparse matrix $\mathbf{M}_J$.

The wavelet formulation of the source vector $\mathbf{s}_J = \mathbf{s}_{f,J} + \mathbf{s}_{v,J}$ is obtained by expanding each element of $\mathbf{s}_{f,J}$ into the set of time samples corresponding to the collocation points. The first time sample of the source vector is replaced by the corresponding initial value.

The final linear circuit equation combining the linear subnetwork and the sources is

$$\mathbf{M}_J\hat{\mathbf{u}}_J = \mathbf{s}_{f,J} + \mathbf{s}_{v,J} \quad (6)$$

where $\hat{\mathbf{u}}_J$ is the vector of the wavelet coefficients of the unknown circuit variables.

B. Nonlinear Error Function

The error function of an arbitrary circuit is developed using connectivity information (described by an incidence matrix and constitutive relations describing the nonlinear elements). The incidence matrix $\mathbf{T}$, is built as described in [10]. This matrix is sparse and moreover, the number of nonzero elements (either $+1$ or -1) is at most twice the number of state variables (n_s). Then $\mathbf{T}$ is expanded by replacing each $+1$ by $\mathbf{W}_J$, and each -1 by $-\mathbf{W}_J$ (in both cases $\mathbf{W}_J$ is reduced by removing the first column and the first row is replaced by zeros). This matrix is denoted $\mathbf{T}_{2,J}$. The transpose of $\mathbf{T}$ is similarly expanded by replacing each $+1$ by a matrix $\mathbf{I}_r$, and each -1 by $-\mathbf{I}_r$, where $\mathbf{I}_r$ is an identity matrix of size $m \times m$ reduced by removing the first row. This matrix is denoted $\mathbf{T}_{1,J}$.

$\mathbf{T}_{1,J}$ and $\mathbf{T}_{2,J}$ capture the connectivity of the nonlinear elements. The nonlinear subnetwork is described by the following generalized parametric equations [12]:

$$\mathbf{v}_{NL}(t) = u[\mathbf{x}(t), \frac{d\mathbf{x}}{dt}, \ldots, \frac{d^n\mathbf{x}}{dt^n}, \mathbf{x}_D(t)] \quad (7)$$

$$\mathbf{i}_{NL}(t) = w[\mathbf{x}(t), \frac{d\mathbf{x}}{dt}, \ldots, \frac{d^n\mathbf{x}}{dt^n}, \mathbf{x}_D(t)] \quad (8)$$

where $\mathbf{v}_{NL}(t)$ and $\mathbf{i}_{NL}(t)$ are vectors of voltages and currents at the common ports, $\mathbf{x}(t)$ is a vector of state variables and $\mathbf{x}_D(t)$ is a vector of time-delayed state variables, *i.e.*, $x_{Di}(t) = x_i(t - \tau_i)$.

Let $\mathbf{x}_J$ be the state variable vector at all collocation points and $\hat{\mathbf{x}}_J$ the corresponding vector of coefficients in the transform domain. The first transform coefficient is excluded from the set of unknowns since it can be derived from the initial condition. Then we denote $\mathbf{v}_{NL,J}(\hat{\mathbf{x}})_J$ and $\mathbf{i}_{NL,J}(\hat{\mathbf{x}}_J)$ the vectors of voltages and currents at the ports of the nonlinear devices at all the collocation points but the first. The error function $\mathbf{F}(\hat{\mathbf{x}}_J)$ is then

$$\mathbf{F}(\hat{\mathbf{x}}_J) = \mathbf{T}_{2,J}\mathbf{M}_J^{-1}\mathbf{s}_{f,J} + \mathbf{T}_{2,J}\mathbf{M}_J^{-1}\mathbf{T}_{1,J}\mathbf{i}_{NL,J}(\hat{\mathbf{x}}_J) - \mathbf{v}_{NL,J}(\hat{\mathbf{x}}_J) \quad (9)$$

which can be expressed as

$$\mathbf{F}(\hat{\mathbf{x}}_J) = \mathbf{s}_{sv,J} + \mathbf{M}_{sv,J}\mathbf{i}_{NL,J}(\hat{\mathbf{x}}_J) - \mathbf{v}_{NL,J}(\hat{\mathbf{x}}_J) \quad (10)$$

Here $\mathbf{s}_{sv,J}$ is the compressed source vector (the initial conditions of the entire linear subcircuit are embedded in it) and $\mathbf{M}_{sv,J}$ is the compressed impedance matrix. Our implementation solves the system of nonlinear equations using a globally convergent quasi-Newton method. The size of $\mathbf{M}_{sv,J}$ is $(m-1)n_s \times (m-1)n_s$, where m is the number of collocation points.

IV. Discussion

Consider the modeling of the 47-section nonlinear transmission line described in [13] (see Fig. 1). Modeling this structure is regarded by many in the field as an extreme test of the performance of transient and steady-state simulators. Fig. 2 compares the results using wavelet transient and Spice3f5 simulations. The small difference in the response is due to slightly different treatments of the diode model in Transim and Spice. A large number of time windows (see [7]) was needed to reduce the number of unknowns to be solved simultaneously. There are clear tradeoffs involved. In wavelet transient analysis the error is minimized over a time interval and there are many more unknowns than when the error is minimized at a single

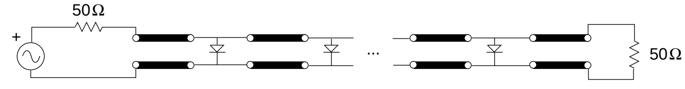

Fig. 1. Model of the nonlinear transmission line.

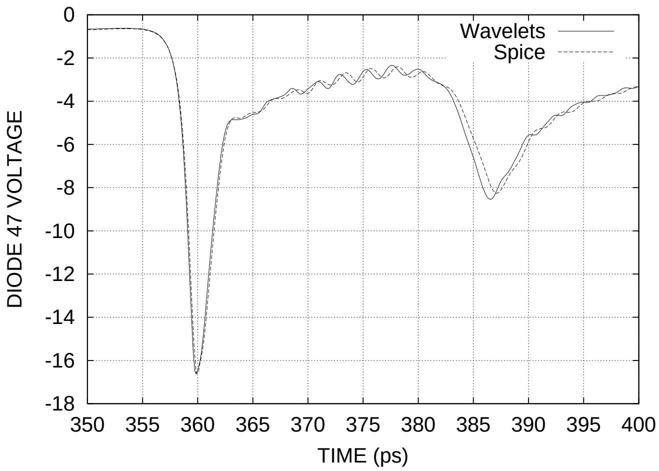

Fig. 2. Comparison of the voltage of the last diode of the nonlinear transmission line.

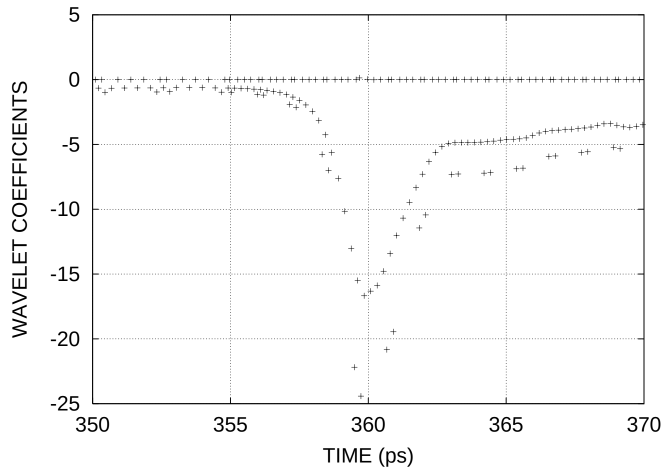

Fig. 3. Coefficients of the state variable of the last diode.

time point. However since error is minimized over a range and because of the $O(h^4)$ convergence rate of the wavelet basis used [6], where h is the time step, there are generally many fewer time points than required in a Spice-like analysis. Table I compares different key aspects of the two

TABLE I
Comparison of Spice and wavelet transient simulations of the first 400 ps transient response of the soliton line.

	Wavelets	Spice
Time (minutes)	50	3
Memory (MB)	41	37
Scalar Unknowns	564	3027
No. of Windows / Time Samples	94	1055
Average Newton Iterations	30	4

simulation methods. Despite the smaller number of scalar unknowns in our state variable wavelet formulation, the overall simulation time is longer than the Spice time because of two reasons: the Jacobian of the nonlinear system of equations in the wavelet method is dense and the initial guess for the nonlinear system at each time window is not as good as in the Spice method. This implies more and slower Newton iterations and therefore more computation time. The lack of a good initial guess also affects the robustness of the analysis. Fig. 3 shows the wavelet coefficients for the state variable of the last diode. Many of these are negligible and could be removed from the calculation by using an adaptive scheme to increase the efficiency of the method. More development is needed before this method reaches its full potential. In particular dynamic variation of resolution including variable resolution at different circuit nodes is required.

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