Magnetic Resonance Imaging
20.4–2

NEW APPLICATIONS OF MEAN FIELD ANNEALING TO THE RESTORATION OF MEDICAL IMAGES

Youn-Sik Han[†,*], Wesley E. Snyder[†]

[†]Department of Radiology
Bowman Gray School of Medicine
Wake Forest University, Winston-Salem, NC 27157-1022

[*]Department of Electrical and Computer Engineering
North Carolina State University, Raleigh, NC 27695

ABSTRACT

This paper presents some new applications of Mean Field Annealing (MFA), a technique which usually finds global minima in nonconvex optimization problems, to the restoration of nuclear medicine and magnetic resonance images. The image restoration technique presented in this paper preserves the edges while reducing noise. By selection of proper parameters, it can be used for segmentation. Nonuniform temperature annealing is introduced for the restoration of images with signal dependent noise. good for 3-D images. By the use of successive over-relaxation (SOR) algorithm in minimization, These MFA image restoration techniques are fast enough for practical use in processing 3-D images.

INTRODUCTION

Bayesian methods are being widely used in the various fields of the image processing [8]. Geman and Geman noted the equivalence of Gibbs distributions to Markov Random Fields and used this equivalence to develop a stochastic relaxation method for the maximum a posteriori (MAP) estimation problem [5].

This Stochastic Simulated Annealing (SSA) technique provided a new tool for use in the field of image restoration, but has some disadvantages. Some of the disadvantages of SSA have been overcome by advances in the treatment of discontinuity and continuously valued images [1, 4]. Recently Bilbro and Snyder [2, 3] developed the so called "Mean Field Annealing" (MFA) technique which approximates stochastic relaxation but is a deterministic method. In this paper we briefly introduce the MFA restoration algorithm and show recent results in its application to the areas of cardiac MRI and nuclear medicine.

MFA IN IMAGE RESTORATION

Since it is reasonable to represent an image by a multidimensional Markov random field, using the equivalence of MRF's to the Gibbs distribution [5], and assuming that the noise is white and Gaussian [3, 5, 6], the maximum a-posteriori probability (MAP) problem becomes a minimization problem of energy function which is can be given by

$$H(f) = \sum_i \left(\frac{(f_i - g_i)^2}{2\sigma_n^2} + b\frac{V(f_i)}{T} \right) \tag{1}$$

where b is a relative weighting factor between the first, "noise" term and prior term, $V(f_i)$, which describes the penalty of an unlikely configuration. Here, σ_n is the standard deviation of the noise, i denotes all the pixels in the image and T denotes the prior model temperature (from SSA). Here, we choose $V(f_i)$ to have a Gaussian functional form for a piecewise-uniform prior model as follows

$$V(f_i) = -\sum_{j \in \aleph_i} \frac{r_{ij}}{\sqrt{2\pi\tau}} exp\left(-\frac{(f_i - f_j)^2}{2\tau}\right) \tag{2}$$

where $\aleph_i$ denotes a set of neighbor pixels of pixel i, r_{ij} is used for discriminating the effect of varying pixel distances, and τ can be considered as a soft threshold for smoothness. If we choose very small τ, the resultant restored image becomes locally homogeneous. (See [6] for a derivation of this approach). Since the pixel values of medical images, such as MR images or nuclear medicine images, often represents a density, an assumption of local homogeneity (small τ) may often be used.

Since $H(f)$ has many local minima, we apply mean field annealing technique (for detail refer to [2, 7]) which converts the problem of minimization on f into the problem of minimization on x which is the mean field approximation of f. The energy equation becomes

$$H(x) = \sum_i H(x_i) \tag{3}$$

and

$$H(x_i) = \frac{(x_i - g_i)^2}{2\sigma_n^2} - b\sum_{j \in \aleph_i} \frac{r_{ij}}{\sqrt{2\pi T'}} exp\left(-\frac{(x_i - x_j)^2}{2T'}\right) \tag{4}$$

Comparing Equations (1) ~ (4), we note that the temperature T moved into the Gaussian form and T' $(= \tau + T)$ now fills the role of threshold and controls the homogeneity of the local region at temperature T.

MFA RESTORATION ALGORITHM

The algorithm of MFA is similar to SSA, but it is a deterministic relaxation at each temperature T:

1.Initialize $x = g$ (corresponds to $T_{init} = \infty$) and $T = T_{init}$

2.Minimize $H(x)$

3.Reduce T

4.If $T > T_{final}$ $(= \tau)$ go to step 2

For faster convergence in each stage of minimization, SOR algorithm is most suitable:

$$x_i^{(n+1)} = x_i^{(n)} - \frac{\omega}{M_i}\frac{\partial}{\partial x_i} H(x_i) \tag{5}$$

where

CH3068-4/91/0000-0097 $01.00 © 1991 IEEE

$$M_i = \frac{\partial^2}{\partial x_i^2} H(x_i)\Big|_{max} = \frac{1}{\sigma_n^2} + b \sum_{j \in \aleph_i} \frac{r_{ij}}{\sqrt{2\pi T^3}} \qquad (6)$$

and $\omega \in (0, 2)$ is the relaxation factor.

The temperature T_{init} can be interpreted as a "melting point", where most of the noise whose standard deviation is around $\sqrt{T_{init}}$ are removed. When the difference between neighboring areas of the image is larger than $2\sqrt{T}$, it is considered that there exists a discontinuity at temperature T, and edges are then preserved.

When noise is dependent on the signal, (for example, in the case of nuclear medicine images, the Poisson process may be approximated by noise variances which are dependent on signal intensity), we can apply a non-uniform initial temperature. In such a case, the local initial temperatures are determined by direct calculation of local variance or mean, and we then apply the same cooling ratio all over the image during annealing. Local initial temperatures can be calculated as

$$T_{init_i} = \frac{\alpha}{N} \sum_{j \in \aleph_i} (g_j - \overline{g}_i)^2 \qquad (7)$$

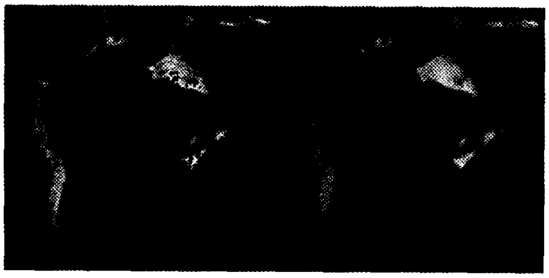
where α is a constant and

$$\overline{g}_i = \frac{1}{N} \sum_{j \in \aleph_i} g_j \qquad (8)$$

RESULTS

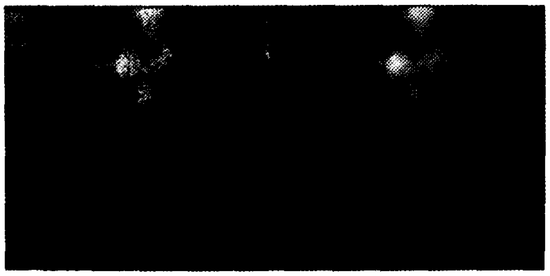
Picture 1. and Picture 2. show the results of the MFA restoration of the Magnetic Resonance cardiac images with uniform temperature, which are done for the segmentation of coronary arteries.

Picture 1. Restoration of cardiac MR image ($T_{init}=\sigma_n=b=25$, $\tau=0.05$, cooling ratio=0.5)



Picture 2. Restoration of cardiac MR image ($T_{init}=\sigma_n=b=100$, $\tau=0.05$, cooling ratio=0.5)

Picture 3. shows the result of non-uniform temperature restoration of a nuclear medicine image which shows a region of cancer of the sternum.Minimization was done at 10 temperatures with cooling rate 0.5.



Picture 3. Restoration of nuclear medicine image with non-unform temperature MFA (10 minimization)

For each restoration of a 256 × 256 image of float data type, 20 ~ 100 seconds were required on a DEC RISC machine.

CONCLUSION

In this paper, we introduced an image restoration technique using Mean Field Annealing and showed its results on medical images. By using an SOR algorithm in the minimization stage, it became fast enough for practical use in 3-D image processing. We showed also a new non-uniform temperature annealing technique which is necessary for the restoration of images of signal dependent noise.

REFERENCES

[1] J. Besag, "On the statistical analysis of dirty pictures", *J. of Royal Statistical Society B*, vol. 48, no. 3, pp. 259-302, 1986.

[2] G. L. Bilbro, W. E. Snyder D. E. Van den Vout, T. K. Miller and M. W. White, "Optimization by Mean Field Annealing", in D. S. Touretzky ed., *Advances in Neural Information Processing*, pp. 91-98, Morgan Kaufmann, 1989.

[3] G. L. Bilbro and W. E. Snyder, "Range image restoration using Mean Field Annealing", in D. S. Touretzky ed., *Advances in Neural Information Processing*, pp. 594-601, Morgan Kaufmann, 1989.

[4] A. Blake and A. Zisserman, *Visual Reconstruction*, The MIT Press, 1987.

[5] D. Geman and S. Geman, "Stochastic relaxation, Gibbs distributions, and the Bayesian restoration of images", *IEEE Trans. on PAMI* vol. PAMI-6, no. 6, pp. 721-741: Nov. 1984.

[6] H. Hiriyannaiah, G.L. Bilbro, W.E. Snyder, and R.C. Mann, "Restoration of Piecewise Constant Images via Mean Field Annealing", *Journal of the Optical Society of America A*, pp. 1701-1912, December, 1989.

[7] H. Hiriyannaiah, *Signal Reconstruction using Mean Field Annealing*, Ph.D. Thesis, North Carolina State University, Raleigh, 1990.

[8] B.R. Hunt, "The application of constrained least squares estimation to image restoration by digital computer", *IEEE Trans. on Computers*, vol. C-26, no. 6, pp. 252-266, 1977.

Corresponding Address:
Youn-Sik Han
Medical Center (BGSM)
Medical Center Boulevard
Winston-Salem, NC 27157-1022
(919) 748-3260

CH3068-4/91/0000-0098 $01.00 © 1991 IEEE