

# MAP Transmission Image Reconstruction via Mean Field Annealing for Segmented Attenuation Correction of PET Imaging

Cliff X. Wang,<sup>1,2</sup> Wesley E. Snyder<sup>1,2</sup>

<sup>1</sup>CCSP, Dept. of ECE, North Carolina State University, Raleigh, NC 27695, USA  
<sup>2</sup>MRI Research Center, Bowman Gray School of Medicine, Winston-Salem, NC 27157, USA

## Abstract

In PET studies, accurate attenuation correction is crucial in quantitative studies. We propose to use a MAP reconstruction algorithm based on mean field annealing technique to reconstruct the transmission image for segmented attenuation correction. This reconstruction method combines both the measured projection data and the *a-priori* knowledge of the property of the image to be reconstructed. A piecewise-constant prior is assumed such that the reconstructed transmission image consists of smooth homogeneous regions separated by sharp edges. By using this approach, the reconstructed image is less noisy and has sharper edges preserved in comparison with the filtered back projection (FBP) method. As a result, segmentation of the transmission image is more accurate. By using this calculated attenuation correction approach, the final reconstructed emission image has less statistical noise than the image corrected by the measured attenuation data.

Keyword: PET Imaging, Attenuation correction, MAP reconstruction, Mean field annealing

## I. INTRODUCTION

Positron emission tomography (PET) is a nuclear medicine imaging modality used to examine the metabolic activity in the body. In quantitative PET studies, accurate attenuation correction is essential because the measured emission data are modulated by the attenuation of the body. The latest development in attenuation correction uses a segmented attenuation correction approach [1],[2],[3],[4] to reduce transmission scan time and to improve final emission image quality. In this approach, a short transmission scan is acquired and the background corrected transmission image is reconstructed. Based on the human anatomy, the transmission image is segmented into different anatomical regions and each region is assigned its theoretical attenuation value. The segmented transmission image is then reprojected to correct the emission scan. Although the scan time and patient dose are reduced drastically, a short transmission scan typically suffers from considerable statistical noise because of low data counts. The filtered back projection (FBP) method may not handle the effects of noise well [5]. As a result, the FBP reconstructed transmission image may be too noisy for a successful segmentation. In this paper, we propose a MAP-based iterative method using mean field annealing for transmission image reconstruction. Based on the observation that a typical transmission image is composed of several homogeneous regions of different anatomical parts and the background, we assume the *a-priori* knowledge of the reconstructed image to be piecewise uniform. Combining both the measured scan data and the prior knowledge, the reconstruction is carried out iteratively. After reconstruction, the less noisy transmission image can be successfully segmented and reprojected to correct the emission scan.

## II. METHOD

In our experiment, a maximum *a-posteriori* (MAP) reconstruction technique is used. Let  $f$  represent the unknown source distribution image to be reconstructed and  $g$  represent the measured projection data. Then  $p(f|g)$  stands for the probability of the reconstructed source distribution image for the given measured projection data. By applying the Bayes' law, we can rewrite  $p(f|g)$  as follows,

$$p(f|g) = \frac{p(g|f)p(f)}{p(g)} \quad (1)$$

where  $p(g|f)$  is the probability of the given projection data for the current estimate of the distribution image,  $p(g)$  is the probability distribution of the projection data, and  $p(f)$  represents the *a-priori* knowledge of the source distribution image  $f$ . In nuclear medicine, the projection data follow the Poisson distribution. The conditional probability  $p(g|f)$  can be written by the definition of Poisson random process,

$$p(g|f) = \prod_i \frac{\exp\left(-\sum_j a_{ij}f_j\right) \left(\sum_j a_{ij}f_j\right)^{g_i}}{g_i!} \quad (2)$$

where  $g_i$  is the number of annihilation counts detected at the detector pair  $i$  and  $f_j$  is the estimate of the source density at the pixel location  $j$ . The probability  $p(g|f)$  determines the likelihood of the projected data with respect to the original source distribution.

We want to get the best reconstructed source distribution such that the *a-posteriori* probability  $p(f|g)$  is maximized. Since the probability distribution of the projection data  $p(g)$  is independent of the source distribution image, it can be dropped from the maximization process. By taking the natural logarithm of the numerator of Equation 1 and changing the sign, maximizing the *a-posteriori* probability is equivalent to minimizing the following new objective function,

$$H_T(f|g) = H_n(g|f) + \beta H_p(f) \quad (3)$$

where  $H_n(g|f) = \log(p(g|f))$  and  $H_p(f) = \log(p(f))$ .

We model the PET image by a Markov random field and the *a-priori* probability has the form

$$p(f) = \frac{1}{Z} \exp(\beta H_p(f)) \quad (4)$$

$H_p(f)$  is as follows,

$$H_p(f) = \sum_i \sum_{j \in \mathcal{N}_i} b_{ij} V(f_i, f_j) \quad (5)$$

where  $V(f_i, f_j)$  is some problem-dependent *a-priori* potential function. This function can be interpreted as a penalty which reflects the

*a-priori* knowledge about each pixel,  $i$ , and its neighborhood,  $\mathfrak{N}_i$ . Based on experience in previous research[5],[6], we use a bounded prior term which has the following form:

$$H_p = \sum_i \sum_{j \in \mathfrak{N}_i} V(f_i, f_j) = \sum_i \sum_{j \in \mathfrak{N}_i} \frac{-b}{\tau} \exp\left(\frac{-\Lambda(f_i, f_j)}{2\tau^2}\right) \quad (6)$$

where  $\Lambda(f_i, f_j)$  is a measure of the similarity between a pixel and its neighbors. The transmission image is typically composed of nearly uniform regions and nearly uniform background. This assumption leads to the selection of a prior term in the form of  $\Lambda = (f_i - f_j)^2$ . This corresponds to a first derivative term and it promotes the reconstructed image to have spatially homogeneous regions separated by sharp step boundaries. The term “annealing” refers to the minimization process which starts with a large  $\tau$ , but gradually reduces  $\tau$  over the applications of the descent step. Refer to reference [6] for details of the annealing method. The minimization is carried out iteratively by using the gradient descent of  $H_T(f)$ . Each pixel is updated according to the following schedule:

$$x_i^{k+1} = x_i^k - \omega \times \frac{\frac{\partial}{\partial x_i} H_T(x)}{\left. \frac{\partial^2}{\partial x_i^2} H_T(x) \right|_{max}} \quad (7)$$

To compare the accuracy of the proposed MAP based reconstruction method with the FBP reconstruction, we created a synthesized image which contained two half discs of greylevel 128 and 256. The background greylevel was 0. Using the PET simulation software developed in our group, we generated the 100K, 500K, and 1M count transmission sinogram of this image. Reconstruction of the images was carried out by using the filtered back projection method, the maximum likelihood method, and the maximum *a-posteriori* method proposed in this paper. Since the true distribution of the simulated source image was known, the mean square errors were calculated between the reconstructed images and the original source image to compare the reconstruction methods.

A chest phantom was used to verify this segmented attenuation correction method. We obtained 3 scan data sets of 1, 10, and 100 million counts. The segmented attenuation correction approach using the proposed method for transmission image reconstruction was used to correct the emission data. As a comparison, the same emission data was corrected by using the conventional method.

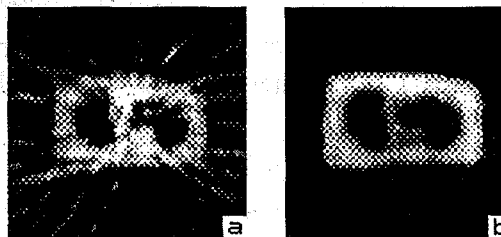
### III. RESULTS

The mean square errors for the reconstructed disc images of 100k, 500k, and 1M counts are tabulated in the Table 1. The mean square errors are normalized by the maximum greylevel(256).

**Table 1: The Normalized Mean Square Error**

Count	FBP No Filter	FBP Hann	ML 30 Iteration	MAP
100K	0.195	0.180	0.1617	0.1400
500K	0.185	0.108	0.0870	0.00791
1M	0.185	0.088	0.0762	0.00747

Figure 1 contains the reconstructed emission image of the chest phantom. Conventional attenuation correction using measured attenuation data was used in the left image. The right image used the proposed segmented attenuation correction using MAP transmission image reconstruction.



**Figure 1. FBP reconstructed final emission image of chest phantom with (a) conventional attenuation correction (Hann filter applied); (b) segmented attenuation correction (Hann filter applied).**

### IV. DISCUSSION

The use of MAP based algorithm has an obvious advantages over filtered back projection method in transmission image reconstruction. The FBP reconstructed transmission image suffers from the noisy low count data. Iterative methods based on maximum *a-posteriori* reconstruction provides improved image quality, which leads to more precise segmentation result. From Table 1, it can be observed that MAP based reconstruction generates smaller MSE than the FBP method. By incorporating the image model as the *a-priori* knowledge into the reconstruction, we can control the final image appearance. A piecewise-linear model is used in the reconstruction so that the final reconstructed image consists of homogeneous regions separated by sharp edges. Experiment results reveal the effectiveness of this approach. The emission image with segmented attenuation correction appears much better than that generated from the conventional attenuation correction method.

### REFERENCES

- 1) Xu, E.Z., Mullani, N.A., Gould, K.L., and Anderson, W.L., “A segmented attenuation correction for PET”, *J. Nuclear Medicine*, 32:161-165
- 2) Hoffman, E.J., Phelps, M.E., “Positron emission Tomography: principles and quantitation”, in *Positron Emission Tomography and Autoradiography: Principles and Applications for the Brain and Heart*, Raven Press, New York, 1986
- 3) Xu, M., Luk, W.K., Culter, P.D., and Digby, W.M., “Local threshold for segmented attenuation correction of PET imaging of the thorax”, *Proceedings of Nuclear Science symposium/Medical Imaging conference*, 3:1862, 1993
- 4) Huang, S.C., Carson, R.E., Phelps, M.E., Hoffman, E.J., Schelbert, H.R., and Kuhl, D.E., “A boundary method for attenuation correction in positron computed tomography”, *J. Nuclear Medicine*, 22:627-637, 1981
- 5) Han, Y.S., “Annealing Methods in Iterative Image Processing with Medical Applications,” Ph.D. Thesis, North Carolina State University, Raleigh, NC, 1993
- 6) Bilbo, G., Snyder, W., “Applying Mean Filed Annealing to Image Noise Removal”, *J. of Neural Network Computing*, p5-17, 1990