Supervised Multispectral Image Segmentation using Active Contours

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Abstract— **Active contours have been widely used as image segmentation methods. The use of level set theory has provided more flexibility and convenience for the implementation of active contours. However, traditional active contour models have some limitations on the segmentation of complicated images whose sub-regions consist of multiple components. The segmentation of multispectral images is even a more difficult problem. We propose an advanced active contour model using the statistics of image intensity based on a multivariate mixture density model. The proposed active contour model shows a robust segmentation capability on the images that traditional segmentation methods cannot properly partition. Numerical experiments with synthetic and real images are presented.**

Index Terms— **active contour, level set, segmentation, multispectral image, multivariate mixture density**

I. Introduction

Segmentation is a process to separate an image into its constituent regions based on a few properties, e.g. intensity, color, or texture. Segmentation algorithms are generally based on either discontinuity among sub-regions, i.e. edges, or uniformity within a sub-region. As the segmentation results depend on the data, autonomous segmentation is one of the most difficult tasks in image analysis. Noise and mixed pixels caused by the poor resolution of sensor images make the segmentation problem even more difficult. In this paper, we propose an advanced segmentation method using a variational framework, called active contours.

Active contours are connectivity-preserving relaxation methods [1], applicable to the image segmentation and motion tracking problems. The basic idea is to start with initial boundary shapes represented in a form of closed curves, i.e. contours:

$$C(s) \equiv \{(x(s), y(s)) : \ 0 \le s \le 1\}, \qquad (1)$$

and iteratively modify them by applying shrink/expansion operations according to the constraints of the image. Those shrink/expansion operations, called contour evolution $\partial C/\partial t$, can be performed by the minimization of an energy function or by the simulation of a geometric partial differential equation (PDE). Active contours have been studied as attractive image segmentation methods because they partition an image into sub-regions with continuous boundaries, while edge detectors based on local filtering, e.g. Canny [2] or Sobel operator, often result in discontinuous boundaries. Also, active contours provide more reasonable segmentation results because they rely on not only the image intensity but also the geometric properties of the sub-regions.

There are two main approaches in active contours based on the mathematic implementation of contour evolution: *classic snakes* and *level sets*. Classic snakes proposed by Kass et al. [3] explicitly move a set of predefined snake points to neighbor positions which minimize the given energy function. Classic snakes provide an accurate location of the edges only if the initial contour is given sufficiently near the edges because they make use of only the local information along the contour. Thus, classic snakes require prior knowledge of the image being segmented. Also, classic snakes cannot detect more than one object simultaneously because the snakes cannot split to multiple boundaries or merge from multiple initial contours. **Level set** approaches proposed by Osher and Sethian [4] move contours implicitly as a particular level, usually the zero level, of a function $\phi(x, y)$ defined on the spatial domain, such as

$$C \equiv \{(x, y) : \phi(x, y) = 0\}, \quad \forall (x, y) \in \Omega \qquad (2)$$

where Ω denotes the entire domain of an image $I(x, y)$. Therefore, the evolution of a level set function $\partial\phi/\partial t$ represents the evolution of a set of contours $\partial C/\partial t$, and the contours C partition the image plane Ω into two subsets $\{\Omega_{in}, \Omega_{out}\}$ according to the sign of the level set function $\phi(x, y)$. The requirement of the initial contour position for level set methods is more flexible than classic snakes. Also, the defined contours can split or merge according to its topological changes as the level set function $\phi(x, y)$ grows or sinks. Therefore, level set methods can detect more than one object simultaneously, and can place multiple initial contours.

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As image segmentation methods, there are two kinds of active contour models according to the force evolving the contours: edge- and region-based.

Edge-based active contours use an edge detector, mostly based on the image gradient, to find the boundaries of sub-regions and to attract the contours to the detected boundaries. *Geodesic active contour* model proposed by Caselles et al. [5], [6] is an example of edge-based active contours. Sapiro proposed *color snakes* [7], [8] for multispectral images $\mathbf{I}(x, y)$ from the geodesic active contour model using a *color gradient* related function using the eigenvalues of a *metric tensor* [9]. The initial contours of edge-based active contours should be placed in completely the inside or the outside of the region of interest (ROI) because the contours move towards only one direction, either inside or outside. Therefore, some level of prior knowledge is still required. Also, edge-based active contours are sensitive to noise due to the image gradient operation.

Region-based active contours measure the uniformity of property within each subset instead of searching geometrical boundaries. Chan and Vese proposed *piecewise-constant active contour* model [10] based on Mumford-Shah segmentation model [11], [12], given by

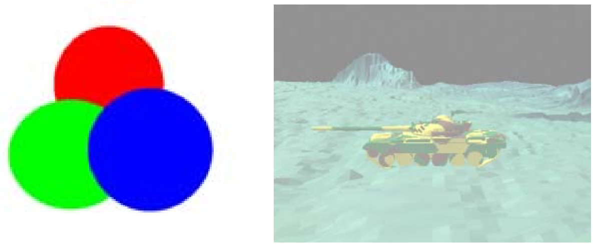
$$\begin{aligned}\frac{\partial \phi(x,y)}{\partial t} &= \delta_\epsilon(\phi(x,y))[\nu\kappa(\phi(x,y)) \\ &\quad - (I(x,y)-\mu_1)^2 + (I(x,y)-\mu_2)^2],\end{aligned} \tag{3}$$

where $\delta_\epsilon(\cdot)$ denotes a regularized form of Dirac delta function, and $\{\mu_1, \mu_2\}$ denotes the mean of the image intensity I measured at the inside and the outside of contours. They extended the same active contour model for the segmentation of multispectral images $\mathbf{I}(x, y)$ [13] by taking the average of differences between image intensity and the mean, measured at each band, given by

$$\begin{aligned}\frac{\partial \phi(x,y)}{\partial t} &= \delta_\epsilon(\phi(x,y))[\nu\kappa(\phi(x,y)) \\ &\quad - \sum_{b=1}^{B}(\mathbf{I}_b(x,y)-\mu_{1b})^2 - (\mathbf{I}_b(x,y)-\mu_{2b})^2],\end{aligned} \tag{4}$$

where $\mathbf{I}_b$ denotes the image intensity at band b, and $\{\mu_{1b}, \mu_{2b}\}$ denote the mean of $\mathbf{I}_b$ measured at the inside and the outside of contours. Equation 3 and 4 consist of two parts: the regularity part, which determines the smoothness of contours using the mean curvature of the level set function $\kappa(\phi(x, y))$, and the energy minimization part, which searches for the image intensity uniform within a subset by minimizing the difference between image intensity and the mean within each subset. Region-based active contours generally do not have any restriction of placing initial contours as they can detect interior boundaries regardless of the position of initial contours. This flexibility and convenience provide a means of an autonomous segmentation method by using a predefined set of initial contours. Also, they are less sensitive to local minima or noise. In this paper, we propose an advanced region-based active contour model based on level set theory.

One of the general assumptions of image segmentation is that each sub-region has a uniform property, such as homogeneous image intensity, as shown in figure 1(a). However, this assumption is not always true in real



(a) (b)

Fig. 1. Assumption and reality: (a) a simple RGB image, (b) a (color) screen shot of a 3D simulator

images. Figure 1(b) shows a screen shot of a simulator where a tank is located in the middle of the scene. Since the tank is covered by a camouflage pattern consisting of multiple colors, traditional segmentation methods based on image intensity cannot separate the tank from the background. Another general assumption of traditional segmentation methods for multispectral images is that a scalar expression can represent vector-valued data, such that brightness or luminance can represent different colors. Equation 4 takes the average of values measured at each band based on this assumption. However, this is also not always true. Each circle in figure 1(a) shows pure red $[255, 0, 0]$, pure green $[0, 255, 0]$, and pure blue $[0, 0, 255]$. Although the averages along RGB channels of these three circles are identical as $255/3$, they clearly have different colors. The goal of the proposed method is to provide robust segmentation results on the multispectral images that traditional segmentation methods cannot properly partition.

II. Segmentation Model

Let the image plane Ω be a disjoint set of subsets and a set of contours, given by

$$\begin{cases}\Omega \equiv (\bigcup_i \Omega_i) \bigcup C \\ \Omega_i \bigcap \Omega_j = \emptyset, \quad \forall i,j, \text{ if } i \neq j\end{cases} \tag{5}$$

Mumford and Shah [11], [12] posed the image segmentation problem as a variational problem to find an optimal piecewise-smooth approximation $f(x, y)$ of the given scalar image $I(x, y)$ and a set of boundaries C, such that the approximation $f(x, y)$ varies smoothly within the connected components of the subsets excluding the boundaries $\Omega \backslash C$. They proposed to solve the variational segmentation problem by minimizing the following global energy function [14]

$$E^{MS}(f,C) \equiv \int_{\Omega} |I(x,y) - f(x,y)|^2 dxdy \qquad (6)$$
$$+ \mu \int_{\Omega \backslash C} |\nabla f(x,y)|^2 dxdy + \nu |C|.$$

The variational boundaries C have the role of approximating the edges of $I(x,y)$ by smoothing $f(x,y)$ only on $\Omega \backslash C$. The minimization of the global energy function approximates the image $I(x,y)$ with $f(x,y)$, smoothes $f(x,y)$, and reduces the length of boundaries $|C|$. The existence and regularity of the solution of the problem above is proven in [11], [15]. The global energy function given in equation 6 can be simplified by ignoring the smoothing term [16] as

$$E(f,C) \equiv \sum_i \int_{\Omega_i} e_i(x,y) dxdy + \nu |C|, \qquad (7)$$

where the objective function $e_i(x,y)$ determines the condition of region-based segmentation for each subset Ω_i such as

$$e_i(x,y) = (I(x,y) - f_i)^2, \quad \forall (x,y) \in \Omega, \ \forall i, \qquad (8)$$

for the piecewise-constant active contours shown in equation 3. The minimum of the objective function is obtained if the approximation f_i is the mean of $I(x,y)$ within the subset. Minimizing the global energy function E does two roles: region-based segmentation by minimizing objective functions $e_i(x,y)$ and smoothing active contours by minimizing the length of contour $|C|$. Combination of these two roles leads to the region-based active contour evolution.

Let the (vector-valued) image intensity $\mathbf{I}$ be a multi-dimensional random variable given by $\mathbf{I} \in \Re^B$ where B denotes the dimension of the image intensity $\mathbf{I}$, which is also equivalent to the number of bands in a multispectral image $\mathbf{I}(x,y)$. We propose an objective function to measure how much an image pixel is likely to be an element of a subset using a **probability density function (PDF) estimated from training samples**. The objective function is given by

$$e_i(x,y) \equiv -\log p_i(\mathbf{I}(x,y)), \quad \forall (x,y) \in \Omega, \ \forall i, \qquad (9)$$

where $p_i(\mathbf{I}) : \Re^B \rightarrow \Re$ denotes the multivariate conditional PDF of a vector-valued image intensity $\mathbf{I}$ on the condition that the image pixel $\mathbf{I}(x,y)$ is an element of the subset Ω_i, given by

$$p_i(\mathbf{I}) \equiv p(\mathbf{I}(x,y) | (x,y) \in \Omega_i), \quad \forall i, \qquad (10)$$

with the unit volume condition

$$\int_{\mathbf{I}} p_i(\mathbf{I}) = 1, \quad \forall i. \qquad (11)$$

Note that $p_i(\mathbf{I})$ is not necessarily a continuous PDF but any PDF-like expression satisfying the conditions above. Based on the energy function shown in equation 7, minimizing the energy function E is equivalent to maximizing the PDF $p_i(\mathbf{I}(x,y))$ for each subset Ω_i.

If a subset Ω_i has training samples given by $\mathcal{I}_i = \{\mathbf{I}_1, \ldots, \mathbf{I}_n, \ldots, \mathbf{I}_N\}$, the PDF of the training samples $\mathcal{I}_i$ is estimated and assigned as the PDF of the image intensity within the subset Ω_i such as

$$p_i(\mathbf{I}) \approx \hat{p}_i(\mathbf{I}) = p(\mathcal{I}_i). \qquad (12)$$

Unfortunately, it is quite rare in practical applications that every subset in the image has training samples. We propose to use a **constant number**, which satisfies the unit volume condition shown in equation 11, as the PDF of unknown subsets. For example, 2^{-8} and $2^{-(8\times3)}$ are proper choices respectively for 8bit scalar (gray) images and 24bit RGB images. The proposed supervised segmentation method still requires some level of prior knowledge because it needs training samples for at least the subset assigned for the target. However, relatively less prior knowledge can provide enough information compared to other existing supervised segmentation methods.

III. Multivariate Mixture Density Model

Region-based segmentation partitions an image looking for the uniformity of statistical property of a subset. If each subset has uniform, i.e. homogeneous, image intensity, a unimodal probability distribution such as a multivariate Gaussian distribution may be good enough as $p(\mathbf{I})$ [16]. However, the statistical property of a subset is often non-uniform, particularly on textured images. Therefore, the use of *multimodal statistical distribution*, often called *mixture density* or *finite mixture*, is necessary to represent the non-uniform statistical property of a subset. We propose to use a **multivariate mixture density function** as the feature representing the statistical property of a subset such that

$$p(\mathbf{I}) \equiv \sum_{k=1}^{K} \alpha_k p(\mathbf{I}|k), \qquad (13)$$

where k denotes a sub-class of the mixture, and $p(\mathbf{I}|k) : \Re^B \rightarrow \Re$ denotes the PDF of the (vector-valued) image intensity $\mathbf{I}$ on the condition that $\mathbf{I}$ is generated by the sub-class k. The weights, often called mixing probabilities, $\{\alpha_k\}$ of each sub-class satisfy

$$\sum_{k=1}^{K} \alpha_k = 1. \qquad (14)$$

There are two approaches to determine the mixture density: parametric and non-parametric.

In parametric approaches, a collection of parameters

$$\mathbf{\Theta} = \begin{bmatrix} \{\alpha_1, \ldots, \alpha_k, \ldots, \alpha_K\}, \\ \{\boldsymbol{\theta}_1, \ldots, \boldsymbol{\theta}_k, \ldots, \boldsymbol{\theta}_K\} \end{bmatrix} \qquad (15)$$

represent a mixture density function $p(\mathbf{I})$, where each $\boldsymbol{\theta}_k$ specify $p(\mathbf{I}|k)$, such that

$$p(\mathbf{I}) \approx p(\mathbf{I}|\boldsymbol{\Theta}) \equiv \sum_{k=1}^{K} \alpha_k p(\mathbf{I}|\boldsymbol{\theta}_k). \tag{16}$$

The parameter set $\boldsymbol{\Theta}$ is estimated from training samples $\mathcal{I}$ based on a particular stochastic model such as a Gaussian distribution $N(\boldsymbol{\mu}, \Sigma)$. For our parametric density estimator, we propose to use a **mixture of multivariate Gaussian distributions** as the parametric PDF $p(\mathbf{I}|\boldsymbol{\Theta})$ of image intensity, given by

$$p(\mathbf{I}|\boldsymbol{\Theta}) \equiv \sum_{k=1}^{K} \alpha_k N(\boldsymbol{\mu}_k, \Sigma_k). \tag{17}$$

The stochastic model above is an arbitrary random distribution whose only prior knowledge is that the statistics of each sub-class k is representable as a weighted multivariate Gaussian distribution $\alpha_k N(\boldsymbol{\mu}_k, \Sigma_k)$. There are four sets of parameters to be estimated: the number of sub-classes K, the weight of each sub-class $\{\alpha_k\}$, the mean vectors of each sub-class $\{\boldsymbol{\mu}_k\}$, and the covariance matrices of each sub-class $\{\Sigma_k\}$. An advanced EM algorithm proposed by Figueiredo and Jain [17] shows a robust estimation of these four sets of parameters without any prior knowledge about the data samples. Although the parametric mixture density model requires long computation time for the learning procedure to estimate the parameters from data samples, it can characterize the statistics of image intensity with just a few variables. It also provides a reasonable estimation results with low number of data samples.

Non-parametric approaches estimate $p(\mathbf{I})$ from the data samples without any assumption of a particular stochastic model. For our non-parametric density estimator, we propose to use a **multi-dimensional histogram density function** as the non-parametric PDF of image intensity because of its simple procedure given by

$$p(\mathbf{I}) \approx h(\mathbf{I}) \equiv \left\{ \frac{1}{N} \frac{\mathrm{hist}[\mathcal{I}]}{\Delta b} \right\}, \tag{18}$$

where $\mathrm{hist}[\cdot]$ denotes a multi-dimensional histogram, Δb denotes the volume of a histogram bin, and N denotes the number of training samples. Although the multi-dimensional histogram requires a huge amount of memory for hyperspectral images, histogram density function requires short computation time because it does not have any learning procedure. Since non-parametric density model does not rely on any particular stochastic model, it will not miss any weak sub-class though the estimated density function may over-fit the data samples, thus be sensitive to noise in training samples.

IV. Active Contour Model

The proposed active contour model is based on the *multi-phase active contour* model proposed by Chan and Vese [14], [18], which can partition an image into more than two subsets simultaneously. Up to $m = 2^J$ subsets can be defined on the entire image plane by a disjoint set of J level set functions. An example of subsets is shown in figure 2 where $\{\Omega_0, \Omega_1, \Omega_2, \Omega_3\}$ denote the four subsets defined by two level set functions $\{\phi_1, \phi_2\}$. By setting the

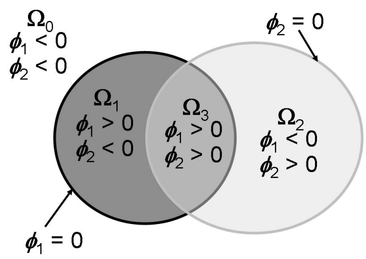


Fig. 2. Subsets and contours defined by multiple level set functions

contour pixels as an element of interior subset, such as

$$\Omega \equiv \bigcup_i \Omega_i, \tag{19}$$

each subset on the spatial domain can be identified by a set of binary *identity functions*

$$\chi_i(x, y) \equiv \left\{ \begin{array}{ll} 1, & \text{if } (x, y) \in \Omega_i \\ 0, & \text{otherwise} \end{array} \right\}, \quad \forall i. \tag{20}$$

composed of a group of regularized unit step functions $\{H_j\}$ given by

$$H_j \equiv H_\epsilon(\phi_j(x, y)) \approx \left\{ \begin{array}{ll} 1, & \text{if } \phi_j(x, y) \geq 0 \\ 0, & \text{if } \phi_j(x, y) < 0 \end{array} \right\}, \tag{21}$$

for $\forall (x, y) \in \Omega$, $\forall j$. The identity functions $\{\chi_i(x, y)\}$ for 2 subsets and 4 subsets are respectively defined as [14]

$$\begin{bmatrix} \chi_0(x, y) \\ \chi_1(x, y) \end{bmatrix} = \left\{ (x, y) : \begin{bmatrix} (1 - H_1) \\ H_1 \end{bmatrix} \right\}, \ \forall (x, y) \in \Omega, \tag{22}$$

and

$$\begin{bmatrix} \chi_0(x, y) \\ \chi_1(x, y) \\ \chi_2(x, y) \\ \chi_3(x, y) \end{bmatrix} = \left\{ (x, y) : \begin{bmatrix} (1 - H_2)(1 - H_1) \\ (1 - H_2)H_1 \\ H_2(1 - H_1) \\ H_2 H_1 \end{bmatrix} \right\}. \tag{23}$$

Using these identity functions, the integration over each subset Ω_i is generalized to the integration over the entire image plane Ω. Also, the length of contours $|C_j|$ is equivalent to the integration of ∇H_j over the image plane where C_j denotes a set of active contours formed by the corresponding level set function $\phi_j(x, y)$. The global energy function of the multi-phase active contour model and the associated Euler-Lagrange equation obtained by minimizing the energy function E with respect to $\Phi = \{\phi_1, \ldots, \phi_j, \ldots, \phi_J\}$ in [14], [18] can be generalized with an arbitrary form of objective functions $e_i(x,y)$, such as

$$E \equiv \sum_{i=0}^{m-1} \int_{\Omega_i} e_i(x,y)dxdy + \nu \sum_{j=1}^{J} |C_j| \quad (24)$$
$$= \sum_{i=0}^{m-1} \int_{\Omega} e_i(x,y)\chi_i(x,y)dxdy + \nu \sum_{j=1}^{J} \int_{\Omega} |\nabla H_j| dxdy,$$

and

$$\frac{\partial \phi_j(x,y)}{\partial t} = \delta_j \left[\nu\kappa_j - \sum_{i=0}^{m-1} e_i(x,y) \frac{\partial \chi_i}{\partial H_j} \right], \quad \forall j, \quad (25)$$

where $\delta_j \equiv \delta_\epsilon(\phi_j(x,y))$, and $\kappa_j \equiv \kappa(\phi_j(x,y))$. The proposed active contour model is obtained by substituting the proposed objective function $e_i(x,y)$ shown in equation 9 into the general multi-phase active contour model shown in equation 25, given by

$$\frac{\partial \phi_j(x,y)}{\partial t} = \delta_j \left[\nu\kappa_j + \sum_{i=0}^{m-1} \log p_i(\mathbf{I}(x,y)) \frac{\partial \chi_i}{\partial H_j} \right]. \quad (26)$$

The actual level set evolution equation for the case of two subsets and four subsets are respectively given by

$$\frac{\partial \phi(x,y)}{\partial t} = \delta_\epsilon(\phi(x,y))[\nu\kappa(\phi(x,y)) + \{\log p_1(\mathbf{I}(x,y)) - \log p_0(\mathbf{I}(x,y))\}], \quad (27)$$

and

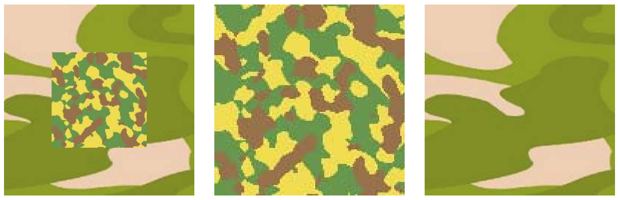
$$\frac{\partial \phi_1(x,y)}{\partial t} = \delta_1 \left[\begin{array}{c} \nu\kappa_1 + \\ \left\{ \begin{array}{c} (\log p_3 - \log p_2)H_2 + \\ (\log p_1 - \log p_0)(1 - H_2) \end{array} \right\} \end{array} \right] \quad (28)$$
$$\frac{\partial \phi_2(x,y)}{\partial t} = \delta_2 \left[\begin{array}{c} \nu\kappa_2 + \\ \left\{ \begin{array}{c} (\log p_3 - \log p_1)H_1 + \\ (\log p_2 - \log p_0)(1 - H_1) \end{array} \right\} \end{array} \right].$$

In the training stage, $p_i(\mathbf{I})$ for known subset is estimated from training samples $\mathcal{I}_i$, and proper constant numbers are assigned to unknown subsets. During the contour evolution, i.e. the test stage, $p_i(\mathbf{I}(x,y))$ returns the probability that the image pixel $\mathbf{I}(x,y)$ is an element of the subset Ω_i. This probability provides a force which changes the corresponding level set function. A similar active contour model using a product of univariate mixture PDF $p_i(I_b)$ instead of multivariate PDF $p_i(\mathbf{I})$ was proposed by the author in [19].

V. Experiments

We present two experiments using the proposed active contour model: segmentation of a synthetic image with a non-parametric density model and segmentation of a real image with a parametric density model.

The object of the first experiment is to extract the rectangular object from the background on the synthetic RGB image ($256 \times 256 \times 3$) shown in figure 3(a). The training samples for the target region is provided, but the background is assumed unknown as is common in automatic target recognition (ATR) problems. Although the shape of the target is simple, this is a difficult segmentation problem because both the target and the background are covered by camouflage patterns consisting of multiple colors. The target region primarily consists of three colors: brown, green, and yellow. The background region primarily consists of different three colors: pink, dark green, and light green. The overall intensity of the two regions have been manipulated such that the mean vectors of both interior and exterior regions are identical as $[R\ G\ B]^{\mathrm{T}} = [163\ 164\ 77]^{\mathrm{T}}$, so traditional segmentation methods based on image intensity, even the vector-valued intensity, cannot produce a desired result. A three-dimensional histogram density function $h(\mathbf{I})$ is estimated from the training samples $\mathcal{I}_{in}$ shown in figure 3(b), and is provided as the non-parametric multivariate mixture density function of the image intensity within the interior subset $p_{in}(\mathbf{I})$. A constant is assigned to the unknown background as $p_{out}(\mathbf{I})$. The proposed active contour model partitions the image into two subsets $\{\Omega_{in}, \Omega_{out}\}$ as shown in figure 4. The proposed active contours successfully



(a) (b) (c)

Fig. 3. A synthetic RGB image: (a) test image ($256 \times 256 \times 3$), (b) known target, (c) unknown background

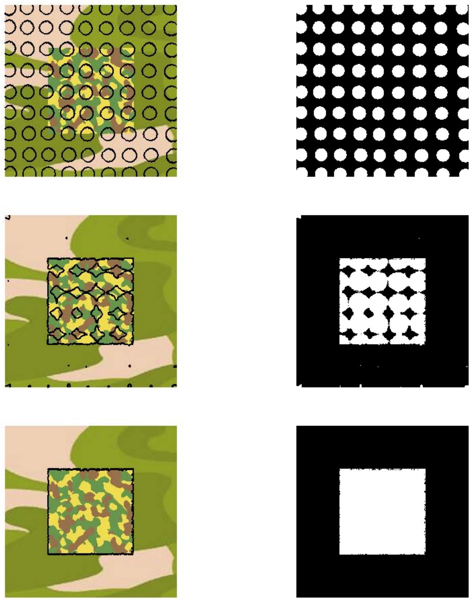


Fig. 4. Segmentation of a synthetic image: (left) contour evolution, (right) corresponding segments

separate the target and the background.

The object of the second experiment is to extract the target region from the background on a real RGB image ($256 \times 192 \times 3$) shown in figure 5(a). A toy tank covered by a camouflage pattern is placed on the carpet. As the first experiment, the training samples for the target region is provided as shown in figure 5(b), but the background is assumed unknown. This is also a difficult segmentation

Fig. 5. A real RGB image: (a) test image ($256 \times 192 \times 3$), (b) training samples

problem because of the camouflage pattern on the target region and the textured background. Most traditional segmentation methods produce an over-segmentation result on the carpet region. The advanced EM algorithm proposed by Figueiredo and Jain [17] estimates a parametric mixture density $p_{in}(\mathbf{I}|\mathbf{\Theta})$ composed of five sub-classes $K = 5$ from the training samples $\mathcal{I}_{in}$. A constant is assigned to the unknown background as $p_{out}(\mathbf{I})$. The proposed active contours successfully separate the target and the background as shown in figure 6.

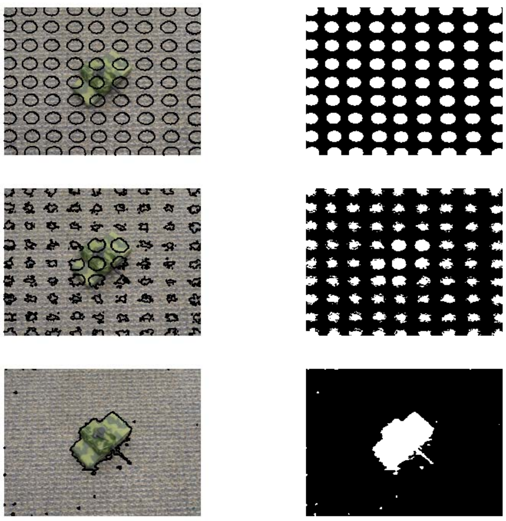


Fig. 6. Segmentation of a real image: (left) contour evolution, (right) corresponding segments

VI. Conclusion

We have proposed an advanced segmentation method using active contours based on level set theory. The proposed active contour model estimates a multivariate mixture density function from training samples using a parametric or a non-parametric density estimation method, and uses it as a measure how likely each image pixel is to be an element of each subset. The proposed supervised segmentation method does not require training samples for all subsets on the image plane. Numerical experiments show robust segmentation results even without training samples for some regions. Although the proposed active contour model is designed for multispectral images, it will also perform well on scalar images.

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