A new beam steering concept: Risley gratings

Chulwoo Oh, Jihwan Kim, John F. Muth, and Michael J. Escuti

North Carolina State Univ, Dept Electrical & Computer Engineering, Raleigh, NC (USA)

ABSTRACT

We introduce a new beam steering concept of the "Risley grating" that consists of independently rotating inline polarization gratings (PGs). The Risley grating concept replaces the bulky prismatic elements of the Risley prisms with thin plates containing polarization gratings, and employs their highly polarization-sensitive diffraction. As rotating two PGs, the output beam tracks within a field-of-regard (FOR), which is determined by the grating period and their relative orientations. Since PGs are typically patterned in thin liquid crystal layers (a few μm thick), the system can be implemented with far less thickness and weight. In addition, these thin gratings can be placed with virtually zero proximity and the beam walk-off becomes negligible. We demonstrate the Risley grating that performs continuous steering with 62° FOR and 89-92% transmittance at 1550 nm wavelength. The governing equations for the steering angles of the Risley grating in the direction cosine space are also presented.

Keywords: Beam Steering, Polarization Grating, Risley Prism, Diffraction

1. INTRODUCTION

The ability of precise beam pointing is crucial to any optical system such as free-space optical communications, countermeasure, laser weapons, and fiber optic switches where beam alignment and target tracking are required. With increasing demands for compact, robust, and cost-effective devices for beam steering, the Risley prisms consisting of pairs of wedge prisms have long been used for its high degree of accuracy and stability. The utilities of the Risley prism, however, are often limited by small steering angles and poor scalability due to bulk prismatic structures. Compact, lightweight beam steering devices comprising of all-thin-plate elements have a number of useful advantages especially for steering where wide angles and large apertures are required.

We introduce the Risley grating, a grating version of the Risley prism, which can perform a continuous beam steering using a pair of polarization gratings (PGs) in independent rotation stages. By replacing wedged prisms with PGs formed in a thin liquid crystal (LC) layer, ultra-compact beam steering devices can be designed for virtually any size of beam. The polarization diffraction properties of PGs provide unique opportunities for beam steering with high throughputs and low levels of side lobes. Several liquid crystal (LC) grating structures (i.e., blazed or binary types) were proposed as a beam steering element.[1–3] The practical use of such LC gratings, however, is limited by their poor angle performance, limited peak efficiency, and low transmittance, and they are not applicable for the Risley gratings. Since two PGs can be placed with a close proximity, the beam walk-off is not an issue and multiple stages can be stacked without increasing the volume.

Here we show the basic concept and its operation principles of this new beam steering device based on rotating PGs, and we report our first demonstration of the Risley grating that performs continuous steering of a laser beam (at 1550 nm) with the field-of-regard (FOR) 62° and 89-92% throughput. The steering angle of the Risley grating is described in the direction cosine space and confirmed by experimental results. Several scanning patterns and side-lobes are also discussed.

Correspondence should be sent to: mjescuti@ncsu.edu, Telephone: +1 919 513 7363

Advanced Wavefront Control: Methods, Devices, and Applications VII, edited by Richard A. Carreras, Troy A. Rhoadarmer, David C. Dayton, Proc. of SPIE Vol. 7466, 74660J · © 2009 SPIE · CCC code: 0277-786X/09/$18 · doi: 10.1117/12.828005

2. BACKGROUND

Polarization gratings have unique diffractive properties due to their spatially variant uniaxial birefringence (Figure 1(a)). Unlike conventional phase or amplitude gratings, the PG operates on modulating local polarization states of incoming light and it leads to remarkable optical performances that include $\sim 100\%$ efficiency into a single diffracted order, circularly polarized first-orders, good angle response, and excellent throughput. The ideal diffraction efficiencies can be derived with the Jones calculus[4,5] as follows

$$\eta_0 = \sin^2\left(\frac{\pi \Delta n d}{\lambda}\right) \tag{1a}$$

$$\eta_{\pm 1} = \frac{1 \mp S_3'}{2}\cos^2\left(\frac{\pi \Delta n d}{\lambda}\right) \tag{1b}$$

where η_m is diffraction efficiency, Δn is the birefringence, d is the grating thickness, λ is the wavelength of incident light, and $S_3' = S_3/S_0$ is the normalized Stokes parameter corresponding to the circular polarization. As shown in Figure 1(b), incoming beam with a circular polarization state (i.e., right-handed) is diffracted from the PG only into one of the first orders and the emerging beam has the orthogonal circular polarization state (i.e., left-handed). The diffraction angle is determined by the grating equation: $\sin\theta_{\pm 1} = \pm\lambda/\Lambda$ for normal incidence. A number of interesting applications have been introduced that are enabled by these unique PG diffraction behaviors in liquid crystal displays, imaging/nonimaging spectropolarimeters, non-mechanical beam steering, and high throughout tunable LC filters.

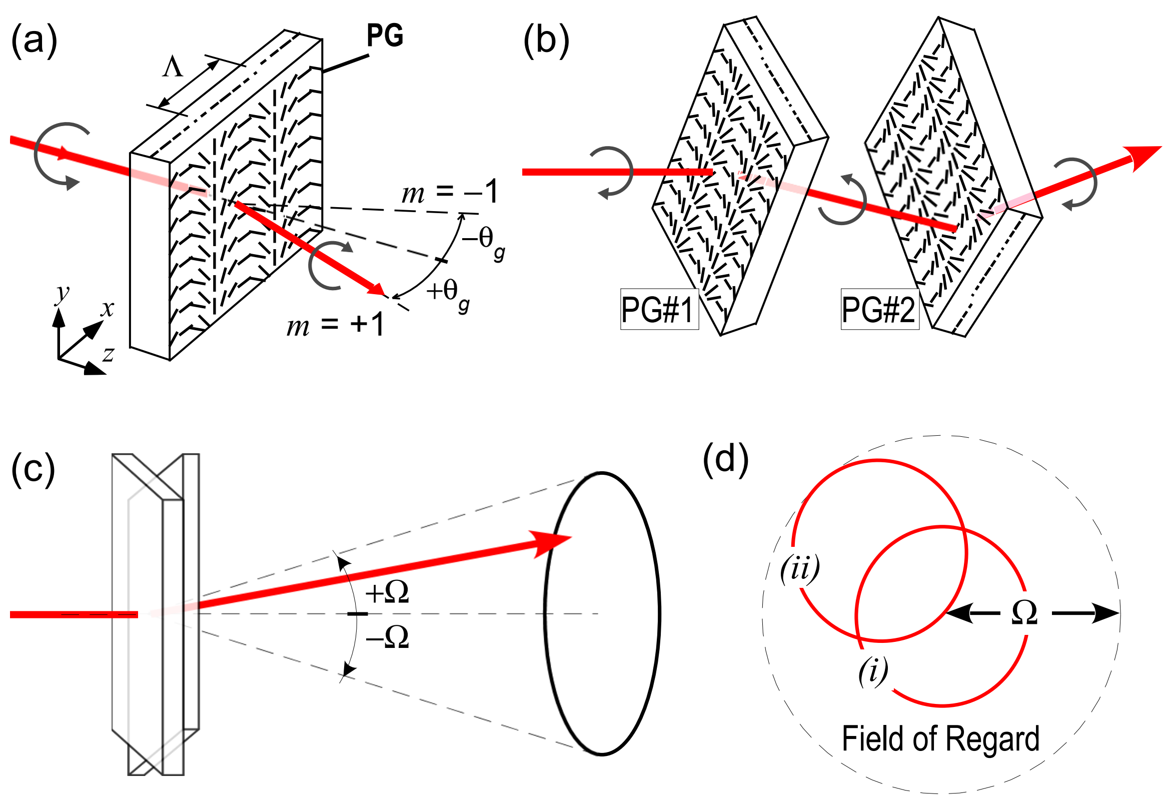

Figure 1. Continuous beam steering by two rotating polarization gratings (PGs), named the "Risley grating": (a) the single-order diffraction from a PG; (b) double diffraction from two PGs with relative orientations; (c) the steered beam from the Risley grating with the maximum steering angle $\Omega = \sin^{-1}(2\lambda/\Lambda)$ (Λ is the period of the PGs); (d) the field of regard of the Risley grating defined as 2Ω and two simple scanning patterns by rotating (i) both PGs with fixed relative orientations and (ii) only one PG with fixed the other PG. Note that the input beam is circularly polarized and the grating is optimized for a half-wave retardation for 100% efficiency ($\Delta nd = \lambda/2$).

Over the last few years, we have experimentally demonstrated high quality PGs with nearly 100% diffraction efficiency using liquid crystal (LC) and photo-alignment[6] materials patterned by polarization holography. We have developed the technology with both highly cross-linked polymer films and electrically switchable LC cells to the extent where we routinely achieve grating periods $\leq 3\ \mu$m,[7,8] for unique and efcient behavior in multiple applications within the visible,[9–11] near-infrared,[5,12] and midwave-infrared.[13] Others have reported also high efficiencies,[14,15] and altogether we show we have advanced beyond the seminal but performance-limited works by Crawford and coworkers.[16] We also demonstrated a prototype nonmechanical beam steering system based on stacked LCPGs and LC waveplates.[12,17]

3. OPERATION PRINCIPLES

Here we consider two identical PGs aligned inline but rotated independently as shown in Figure 1(b). By controlling the relative orientations of the PGs, continuous steering can be achieved described as follows. A circularly polarized, collimated, narrowband beam at normal angle to the first PG is diffracted into one of the first orders depending on the handedness of the circular polarization. The beam direction is determined by the diffraction angle θ_g and the grating orientation ϕ_1. Note that the circular polarization is flipped to the opposite handedness upon the PG diffraction. The second PG (with the same diffraction angle θ_g and the azimuth angle ϕ_2) receives this beam and redirects it to the final steering angle, which is determined by the relative grating orientations. As rotating the PGs, the emerging beam points with angles within the field-of-regard, defined by an angle 2Ω.

The diffraction angle of a single PG at normal incidence is determined only by the grating period Λ and the wavelength λ, which follows the grating equation ($\theta_g = \sin^{-1}(\lambda/\Lambda)$). Diffraction from multiple gratings with different relative orientations, however, is somewhat more complicated because the angle relationship is nonlinear. Therefore, it is often convenient to introduce the direction cosine space where diffraction can be represented as a simple, linear vector.[18] Figure 2(a) shows the diffraction vector $\mathbf{G}_1$ for the first PG oriented at ϕ_1. Then, the beam is diffracted again by the second PG of which diffraction is represented by another vector $\mathbf{G}_2$. The final beam direction can be described as a simple vector sum of two diffraction vectors $\mathbf{G} = \mathbf{G}_1 + \mathbf{G}_2$

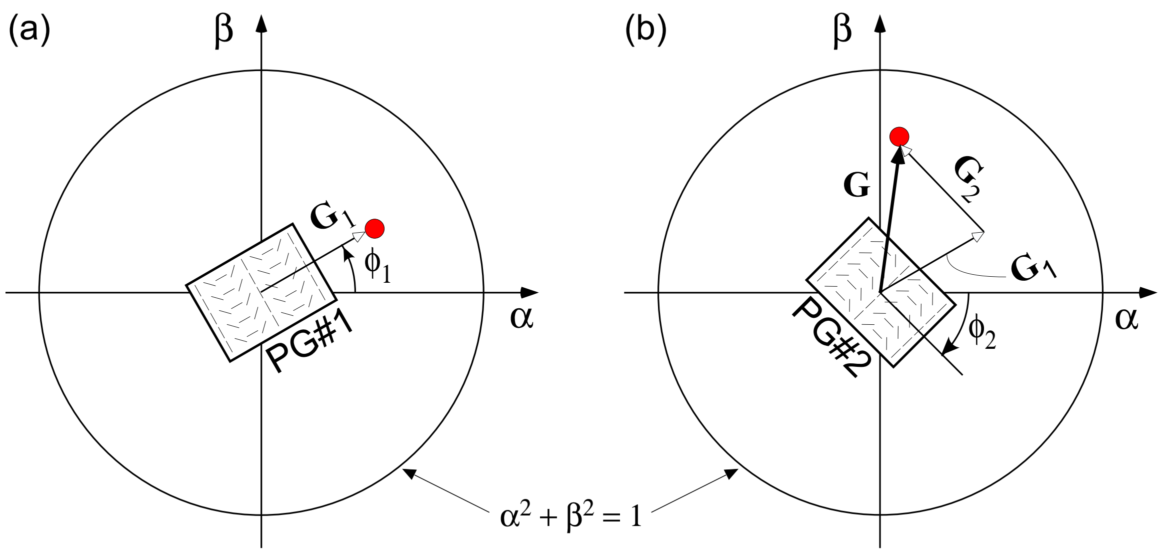

Figure 2. Vector representations of diffraction from two PGs with relative orientations ($\phi_{1,2}$) in the direction cosine space: (a) the first-PG diffraction described as a vector $\mathbf{G}_1$; (b) the second-PG diffraction described as another vector $\mathbf{G}_2$. The final diffraction direction can be described simply by a sum of the diffraction vectors as $\mathbf{G} = \mathbf{G}_1 + \mathbf{G}_2$.

as shown in Figure 2(b). The direction cosines of the steered beam are given by

$$\alpha = \sin(\theta_g)\left[\cos(\phi_1) - \cos(\phi_2)\right] \tag{2a}$$

$$\beta = \sin(\theta_g)\left[\sin(\phi_1) - \sin(\phi_2)\right] \tag{2b}$$

$$\gamma = \sqrt{1 - \alpha^2 - \beta^2}, \tag{2c}$$

where $\phi_{1,2}$ are the grating orientations of the first and second PGs, respectively. We should note that Eqs. (2a)–(2c) are valid only where $\alpha^2 + \beta^2 < 1$. The azimuthal and polar angles of the beam direction can be obtained from the direction cosines as follows

$$\phi = \tan^{-1}(\beta/\alpha) \tag{3a}$$

$$\theta = \cos^{-1}(\gamma) \tag{3b}$$

Steering at the maximum angle Ω ($= \sin^{-1}(2\lambda/\Lambda)$, same to the second-order diffraction angle) occurs when $\phi_2 = \phi_1 + \pi$ or two PGs are aligned in an antiparallel orientation. The field of regard (FOR) is defined as 2Ω, and the steered beam always points a direction within the solid angle of $\pm\Omega$ as shown in Figure 1(c). More on the direction cosine descriptions for the diffraction from two rotating PGs are discussed in Appendix A.

4. EXPERIMENT

We demonstrated the Risley grating beam steering with 62° FOR at 1550 nm using a pair of PGs formed as liquid crystal cells (Λ =6 μm and $\theta_g = 15°$ at 1550 nm). The two PGs were separately mounted in rotation stages and their orientations were controlled to achieve steering of a collimated beam from a infrared laser with circular polarization. We confirmed the steering angle for different grating orientations and measured the beam powers in the forwarding direction.

Defect-free PG samples, formed as liquid crystal cells, were fabricated using polarization holography and photo-alignment materials. In particular, we utilized a linear-photopolymerizable polymer (LPP) ROP-103/2CP (from Rolic) as a photo-alignment material and nematic liquid crystal LCMS-102 (from Boulder Nonlinear Systems, $\Delta n = 0.31$ at 1550 nm). Two glass substrates were coated with LPP and then assembled to make a cell

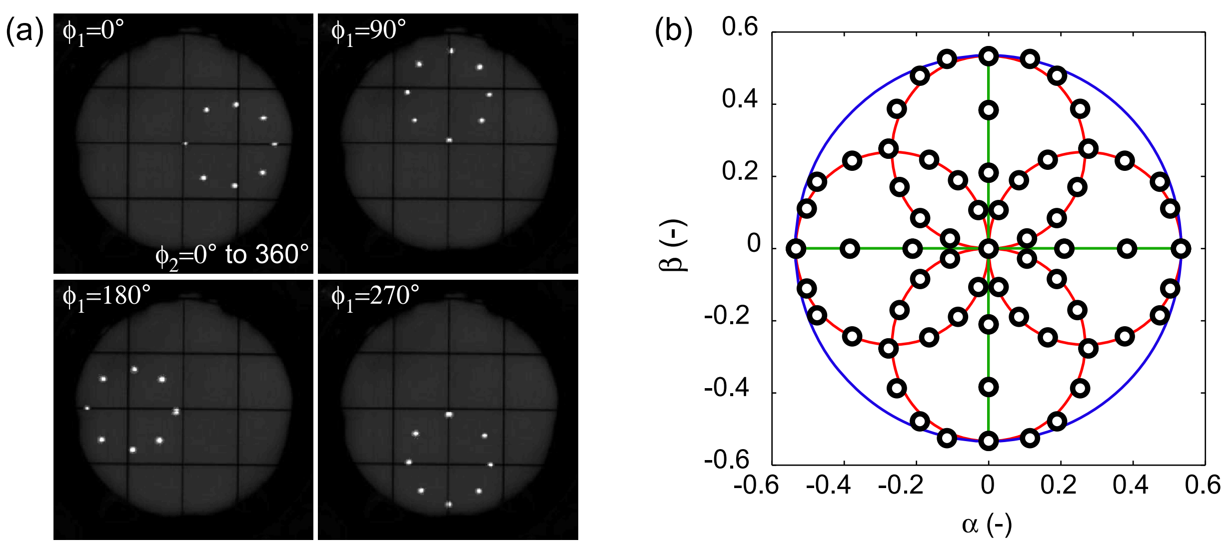

Figure 3. Demonstration of the Risley grating with 62° FOR using two PGs ($\Lambda = 6$ μm): (a) simple scanning patterns (with fixed first-PG orientations and rotating the second PG) of the steered IR laser beam (1550 nm) on an IR sensitive detecting screen; (b) the direction cosines calculated from the measured steering spots (circles), which well matched to the analytical solutions (solid lines) from Eqs. (2a) and (2b). Two PGs were optimized for the maximum efficiency (> 98%) at 1550 nm ($\Delta nd = \lambda/2$, where $\Delta n = 0.31$ and $d \approx 2.5$ μm) and the IR laser was circularly polarized before the first PG.

with 2.5 μm thickness. This cell was exposed with two orthogonal, circularly polarized beams from a HeCd laser (at 325 nm) to record the PG pattern onto the LPP layers. After the UV exposure, the LC material was filled in the isotropic state (at 150°C). The samples exhibit nearly ideal PG diffraction with $> 98\%$ first-order efficiency and no observable higher order. Note that both glass surfaces were laminated with anti-reflection coatings to minimize reflection losses.

Continuous steering of the beam within $\Omega = \pm 31°$ was achieved by individually rotating two PGs with high optical throughput from 89% to 92%. The angles of the steered beam were measured on an IR sensitive detecting screen with different PG orientations. We show simple scanning patterns of the steered beam in Figure 3(a) with

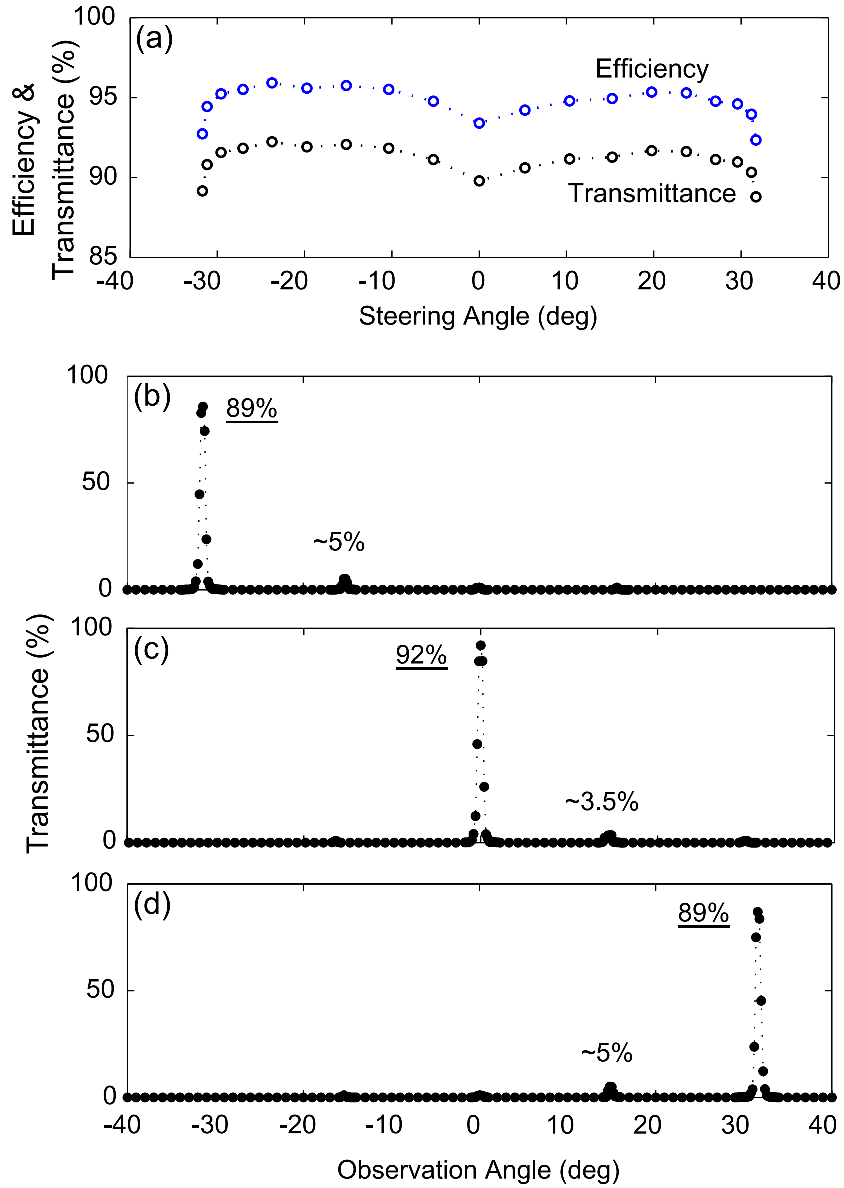

Figure 4. High throughput from 89% to 92% (efficiency from 92% to 96%) of the Risley grating for all steering angles between $\pm 31°$: (a) measured transmittance and efficiency for different steering angles; (b)–(d) power scanning across $\pm 40°$ for three different steering angles of $-30°$, 0, and 30°. Sidelobes in the range of 1% to 6% of the input power were observed at angles that are multiples of the diffraction angle (θ_g), which are diffraction leakages primarily due to oblique incidence to the second PG.

the first PG fixed at $\phi_1 = 0°, 90°, 180°, 270°$ and rotating the second PG ($\phi_2 = 0°$ to $360°$ for each case). The direction cosines corresponding to these angles were calculated as shown in Figure 3(b), which are well matched to calculated angles from Eqs. (2a)–(2c).

We measured the transmitted power for different steering angles and confirmed high throughputs up to 92% transmittance as shown in Figure 4(a) (i.e., transmittance defined as $T = P_{out}/P_{in}$, where P_{in} is the input power and P_{out} is the power in the steered direction). We also calculated the diffraction efficiency (Figure 4(a)) as a normalization that removes the effect of the substrates, defined as $\eta = P_{out}/P_{tot}$, where P_{tot} is the total transmitted power in the forwarding direction. The results show high transmittance from 89% to 92% (efficiency from 92% to 96%) for all steering angles, with some dependency on the steering angle. We estimate roughly 4% of the transmittance loss due to Fresnel-type reflections at the glass/LC interfaces. The remaining loss is due to diffraction leakages that appear in sidelobes.

To characterize sidelobes, we measured the transmitted power by scanning observation angles (±40°) in the forwarding direction. Figures 4(b)-4(d) show measured transmittance for three different steering angles (−31°, 0°, +31°). The level of sidelobes varies in the range of 1% to 6% of the input power, again also depending on the steering angle. Note that, in all cases, the sidelobe was observed at angles that are multiples of θ_g. These leakages primarily result from oblique incidence to the second PG, and can be reduced by the use of higher birefringence LC materials and additional retardation compensation techniques.

5. SUMMARY

We introduced the new beam steering device based on two rotating PGs, named the Risley grating, that can perform continuous steering with a wide FOR and high throughputs. We demonstrated a prototype Risley grating with 62° FOR at 1550 nm wavelength using two identical PGs ($\Lambda = 6$ μm, $>$ 98% first-order efficiency). The Risley grating manifests excellent steering operation with high optical throughput (89% to 92%). Since the PGs are formed in thin liquid crystal layers (a few μm thickness), the Risley grating can be scaled to almost arbitrarily large areas with an ultra-compact and lightweight form factor. Larger steering angles, further loss reduction, and implementation at other wavelengths are certainly possible through continued optimization of substrates and PG materials, as discussed.

Acknowledgment

The authors gratefully acknowledge financial support from the US Air Force Research Laboratory (SBIR Phase II) and the National Science Foundation (ECCS-0621906).

REFERENCES

[1] M. L. Jepsen and H. J. Gerritsen, "Liquid-crystal-filled grratings with high diffraction efficiency," *Opt. Lett.* **21**, pp. 1081–1083, 1996.

[2] X. Wang, D. Wilson, R. Muller, P. Maker, and D. Psaltis, "Liquid-crystal blazed-grating beam deflector," *Appl. Opt.* **39**, pp. 6545–6555, 2000.

[3] Y. Zhang, B. Wang, P. J. Bos, J. Colegrove, and D. B. Chung, "High-efficiency, liquid-crystal-based, controllable diffraction grating," *J. Opt. Soc. Am. A* **22**, pp. 2510–2515, 2005.

[4] J. Tervo and J. Turunen, "Paraxial-domain diffractive elements with 100% efficieincy based on polarization gratings," *Opt. Lett.* **25**, pp. 785–786, 2000.

[5] M. J. Escuti, C. Oh, C. Sanchez, C. W. M. Bastiaansen, and D. J. Broer, "Simplified spectropolarimetry using reactive mesogen polarization gratings," *Proc. SPIE* **6302**, p. 632614, 2006.

[6] M. Schadt, H. Seiberle, and A. Schuster, "Optical patterning of multi-domain liquid crystal displays with wide viewing angles," *Nature* **381**, p. 123102, 1996.

[7] R. K. Komanduri, C. Oh, and M. J. Escuti, "Reflective liquid crystal polarization gratings with high efficiency and small pitch," *Proc. SPIE* **7050**, p. 70500J, 2008.

[8] C. Oh, R. K. Komanduri, B. L. Conover, and M. J. Escuti, "Polarization-independent modulation using standard liquid crystal microdisplays and polymer polarization gratings," *Int. Disp. Res. Conf.* **28**, pp. 298–301, 2008.

[9] M. J. Escuti and W. M. Jones, "A polarization-independent liquid crystal spatial light modulator," *Proc. SPIE* **6332**, p. 63320M, 2006.

[10] R. K. Komanduri, W. M. Jones, C. Oh, and M. J. Escuti, "Polarization-independent modulation for projection displays using small-period LC polarization gratings," *J. SID* **15**, pp. 589–594, 2007.

[11] J. Kim and M. J. Escuti, "Snapshot imaging spectropolarimeter utilizing polarization gratings," *Proc. SPIE* **7086**, p. 708603, 2008.

[12] J. Kim, C. Oh, M. J. Escuti, L. Hosting, and S. Serati, "Wide-angle nonmechanical beam steering using thin liquid crystal polarization gratings," *Proc. SPIE* **7093**, p. 709302, 2008.

[13] C. Packham, M. J. Escuti, G. Boreman, I. Quijano, J. C. Ginn, B. Franklin, D. J. Axon, J. H. Hough, T. J. Jones, P. F. Roche, M. Tamura, C. M. Telesco, N. Levenson, J. M. Rodgers, and J. P. McGuire, "Design of a mid-IR polarimeter for SOFIA," *Proc. SPIE* **7014**, p. 70142H, 2008.

[14] C. Provenzano, P. Pagliusi, and G. Cipparrone, "Highly efficient liquid crystal based diffraction grating induced by polarization holograms at the aligning surfaces," *Appl. Phys. Lett.* **89**, p. 121105, 2006.

[15] S. R. Nersisyan, N. V. Tabiryan, L. Hoke, D. M. Steeves, and B. Kimball, "Polarization insensitive imaging through polarization gratings," *Opt. Express* **17**, pp. 1817–1830, 2009.

[16] G. P. Crawford, J. N. Eakin, M. D. Radcliffe, A. Callan-Jones, and R. A. Pelcovits, "Liquid-crystal diffraction gratings using polarization holography alignment techniques," *J. Appl. Phys.* **98**, p. 123102, 2005.

[17] P. F. McManamon, P. J. Bos, M. J. Escuti, J. Heikenfeld, S. Serati, H. Xie, and E. A. Watson, "A review of phased array steering for narrow-band electrooptical systems," *Proc. IEEE* **97**, pp. 1078–1096, 2009.

[18] J. E. Harvey and C. L. Vernold, "Description of diffraction grating behavior in direction cosine space," *Appl. Opt.* **37**, pp. 8158–8160, 1998.

APPENDIX A. DIRECTION COSINE DESCRIPTIONS FOR DIFFRACTION FROM TWO ROTATING GRATINGS

Diffraction behaviors of gratings can be conveniently described in the direction cosine space. Especially, the conical diffraction is traced by linear relationships between two projected dimensions (α, β). Here we derive the governing equations for the exiting diffraction angle from two rotating inline PGs in the direction cosine space.

An arbitrary vector in the Cartesian coordinate space can be expressed in the direction cosine space as follows

$$\mathbf{A} = \hat{\imath}A_x + \hat{\jmath}A_y + \hat{k}A_x \tag{A-1a}$$

$$= A(\hat{\imath}\alpha + \hat{\jmath}\beta + \hat{k}\gamma) \tag{A-1b}$$

where $A = |\mathbf{A}|$ is the magnitude of the vector. α, β, and γ are projections of the direction vector ($\hat{u}_A = \mathbf{A}A^{-1}$) onto x-, y-, and z-axes, and can be expressed by the angles between the direction vector and $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$ as follows

$$\alpha = A_x/A = cos(\theta_x) \tag{A-2a}$$

$$\beta = A_y/A = cos(\theta_y) \tag{A-2b}$$

$$\gamma = A_z/A = cos(\theta_z) \tag{A-2c}$$

It is convenient to express α, β, and γ with the polar and azimuth angles (θ, ϕ) in the Spherical coordination space because these angles are useful to describe angular behaviors.

$$\alpha = \sin(\theta)\cos(\phi) \tag{A-3a}$$

$$\beta = \sin(\theta)\sin(\phi) \tag{A-3b}$$

$$\gamma = \cos(\theta) \tag{A-3c}$$

Since these direction cosines satisfy $\alpha^2 + \beta^2 + \gamma^2 = 1$, only α and β are sufficient to describe the angle space and γ can be obtained through $\gamma^2 = 1 - \alpha^2 - \beta^2$. From this relationship, we can treat any problem in the α-β space.

The direction cosines for the m-th diffracted order at an arbitrarily oblique angle (θ_{in}, ϕ_{in}) can be written as follows

$$\alpha_m = \alpha_{in} + m\left[\frac{\lambda\cos(\phi)}{\Lambda}\right] = \alpha_{in} + m\Delta\alpha \tag{A-4a}$$

$$\beta_m = \beta_{in} + m\left[\frac{\lambda\sin(\phi)}{\Lambda}\right] = \beta_{in} + m\Delta\beta \tag{A-4b}$$

$$\gamma_m = \sqrt{1-\alpha_m^2-\beta_m^2} \tag{A-4c}$$

where ϕ is the azimuthal angle of the grating orientation and α_{in}, β_{in}, and γ_{in} are given by

$$\alpha_{in} = \sin(\theta_{in})\cos(\phi_{in}) \tag{A-5a}$$

$$\beta_{in} = \sin(\theta_{in})\sin(\phi_{in}) \tag{A-5b}$$

$$\gamma_{in} = \cos(\theta_{in}) \tag{A-5c}$$

The diffraction can be represented as a vector $\mathbf{G}$ defined as $\mathbf{G} = \hat{\alpha}\Delta\alpha + \hat{\beta}\Delta\beta$ ($\hat{\alpha}$ and $\hat{\beta}$ are the unity vectors for α and β, respectively). The direction vector for the m-th diffracted order now can be written as $\mathbf{A}_m = \mathbf{A}_{in} + m\mathbf{G}$.

Now, we can consider a special case of two identical polarization gratings that are aligned inline but rotating around the propagation axis (the z-axis). The orientation angles of each grating from the x-axis are ϕ_1 and ϕ_2, respectively. The grating vectors for the PGs are given by $\mathbf{G}_{1,2} = \hat{\alpha}\Delta\alpha_{1,2} + \hat{\beta}\Delta\beta_{1,2}$, where $\Delta\alpha_{1,2} = (\lambda/\Lambda)\cos(\phi_{1,2})$ and $\Delta\beta_{1,2} = (\lambda/\Lambda)\sin(\phi_{1,2})$.

The direction cosines of the diffracted beam from the first PG can be written as follows

$$\alpha^{(1)} = \alpha_{in} + \Delta\alpha_1 \tag{A-6a}$$

$$\beta^{(1)} = \beta_{in} + \Delta\beta_1 \tag{A-6b}$$

We assume that the incident beam is circularly polarized (i.e., left-handed), which leads to only one of the first-orders (i.e., +1-order) with 100% efficiency and the orthogonal circular polarization (i.e., right-handed). The beam coming from the first PG is diffracted again by the second PG but into the other first-order (i.e., −1-order) because the handedness of circular polarization is flipped by the first PG. The direction cosines of the outgoing beam from the second PG can be written as follows

$$\alpha^{(2)} = \alpha^{(1)} - \Delta\alpha_2 \tag{A-7a}$$

$$\beta^{(2)} = \beta^{(1)} - \Delta\beta_2 \tag{A-7b}$$

For normal incidence ($\alpha_{in} = \beta_{in} = 0$), Eqs. A-7a and A-7b are simplified as

$$\alpha^{(2)} = \Delta\alpha_1 - \Delta\alpha_2 = (\lambda/\Lambda)\left[\cos(\phi_1) - \cos(\phi_2)\right] \tag{A-8a}$$

$$\beta^{(2)} = \Delta\beta_1 - \Delta\beta_2 = (\lambda/\Lambda)\left[\sin(\phi_1) - \sin(\phi_2)\right] \tag{A-8b}$$

and $\gamma^{(2)}$ is obtained as $\gamma^{(2)} = [1 - \left(\alpha^{(2)}\right)^2 - \left(\beta^{(2)}\right)^2]^{1/2}$. Note that $\gamma^{(2)} \geq 0$ because we only consider a hemisphere in the forwarding direction. Finally, we get the angles for the emerging beam as follows

$$\phi = \tan^{-1}(\beta^{(2)}/\alpha^{(2)}) \tag{A-9a}$$

$$\theta = \cos^{-1}(\gamma^{(2)}) \tag{A-9b}$$

The maximum polar angle $\theta_{max} = \Omega = \sin^{-1}(2\lambda/\Lambda)$ is obtained when $\phi_2 = \phi_1 + \pi$, which indicates the second PG is rotated by 180° with respect to the first PG or two PGs are aligned antiparallel. The direction cosines for the maximum angle cases are given by $\alpha^{(2)} = (2\lambda/\Lambda)\cos(\phi_1)$ and $\beta^{(2)} = (2\lambda/\Lambda)\sin(\phi_1)$, which are equivalent to the direction cosines of the second-order diffraction of the first PG.