Jump the Q: A Fast Jitter Tolerance Measurement Method Using Q-Statistical Model

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Abstract—With high-speed receivers and clock data recovery (CDR) blocks operating at speeds in excess of 10 Gbps, stringent CDR jitter tolerance test criteria are necessary to qualify device reliability. A robust CDR jitter tolerance test should accommodate test criteria with extremely low bit error rate (BER) values, usually 10⁻¹² or lower for typical industrial protocols. Using conventional methods, the time required to measure a complete set of CDR jitter tolerance values can stretch into weeks depending on the data rate. In general, measurement time increases tenfold for a similar tenfold reduction in BER. This translates to prohibitively long measurement times for BER values of 10⁻¹⁵ and lower. Statistical extrapolation for low BER measurement such as Q scale has been widely used in the industry, but this is only applied for transmitter measurements, specifically jitter measurements. This paper introduces a novel, fast measurement method for receiver testing based on the Q-statistical method to predict the BER for high-volume data transmission based on small sample data sets. Experimental data using this method show that extrapolated jitter tolerance values for BER values down to 10⁻¹⁵ can achieve an accuracy of 1.25 mUI. This innovative method improves the efficiency of jitter tolerance tests by significantly reducing measurement time. Furthermore, the method allows for the extension of measurement scope to cover previously unattainable jitter tolerance values for lower BER values. With these advantages, full jitter tolerance characterization on Altera Stratix® IV GX devices successfully meets very aggressive product rollout and time-to-market schedule, even with measurements for 24 protocols and support for BER of 10⁻¹² and lower.

I. INTRODUCTION

In the communications industry, the health and quality of a communications system are typically guaranteed through a quality-of-service (QoS) agreement. One of the best ways of gauging the performance of a system is by measuring bit error rate (BER). The BER is a measure of the number of error bits that will occur in a sequence of bits transmitted through a communications network, system or device [3]. In other words, BER is the error probability of any given bit. The fewer the errors measured, the better the performance of the system.

In this paper, BER measurement is used to characterize the clock data recovery (CDR) block jitter tolerance performance to determine compliance with the industry standard protocol specification. This method can be employed on any other I/O protocols such as XAUI, PCIe Gen1, PCIe Gen2, GIGE, and CEI that specify a BER for protocol compliance. Most I/O protocols require a BER in the range of 10⁻¹² to 10⁻¹⁵. A lower BER value translates to a more stringent test criterion. I/O protocols need to operate with fewer errors while thorough system characterization is more challenging and time consuming. Furthermore, aggressive time-to-market schedules in today’s competitive environment necessitate shorter timelines for testing. To overcome these challenges, various statistical analyses have been widely applied for low BER measurements in the industry. But these methods are only applied for the transmitter jitter measurements and not for the receiver jitter tolerance testing [7].

II. BER IN THE INDUSTRY

Many different factors contribute to bit errors in transmitted bits. Bit errors occur when a receiver misinterprets the logic level of bits due to noise, jitter or electromagnetic interference (EMI) [8]. Exploring these factors is the key to understanding BER. For example, total jitter can be decomposed into deterministic jitter (DJ) and random jitter (RJ) [6]. Deterministic jitter consists of intersymbol interference (ISI) and periodic jitter (PJ) from board traces and power supplies. These jitter components affect the BER of a communications system differently [7].

BER is widely used as a measure of a system’s performance. Typically, a test setup to measure the BER consists of a pattern generator that transmits a unique pattern, and a pattern checker. The pattern checker compares the output bit pattern with the pattern applied at the input of the system-under-test [1]. Any discrepancy between the two patterns is flagged as an error bit. Transmitted test patterns are usually generated using the pseudo-random binary sequence (PRBS) generators to emulate standard data traffic and are random sequences of different combinations of 1’s and 0’s. In order to reliably guarantee the performance of a system, a 95% confidence level is selected as the basis for BER measurements in this paper.

Measurement time is dependent on the data rate and the required BER. Common I/O protocols have data rates ranging from 1 to 10 Gbps and BER requirements range from 10⁻¹² to 10⁻¹⁵ [10]. In addition, jitter tolerance is usually measured
across a range of frequencies to determine the frequency response of the system-under-test. Therefore, the measurement time for a complete set of jitter tolerance values can stretch into years depending on conditions as illustrated in Table 1 [2]. Prohibitively long measurement times present a significant challenge to jitter tolerance characterization. In the following sections, this paper will provide an alternative solution.

<table>
<thead>
<tr>
<th>Table 1</th>
<th>TIME FOR ONE ERROR TO OCCUR AT DIFFERENT BIT RATES</th>
</tr>
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<tbody>
<tr>
<td>BER</td>
<td>10^{-4}</td>
</tr>
<tr>
<td>Interlaken 6.375 Gbps</td>
<td>5 us</td>
</tr>
<tr>
<td>Interlaken 3.125 Gbps</td>
<td>10 us</td>
</tr>
<tr>
<td>GIGE 1.25 Gbps</td>
<td>24 us</td>
</tr>
</tbody>
</table>

III. THE Q FACTOR AND THE Q STATISTICAL METHOD

The Q factor, better known as “Q”, is a measure of quality for a system. Q is defined for any signal where its mean values $\mu_0$ and $\mu_1$ and noise power values (variance) $\sigma_0^2$ and $\sigma_1^2$ are sensible regardless of noise model type such as Gaussian, Rayleigh, or Gamma. The $\mu_0$ and $\sigma_0^2$ can be extracted if the probability density function (PDF) for a receiver signal’s low level (level “0”) is definable. Similarly, $\mu_1$ and $\sigma_1^2$ can be obtained based on the PDF for the receiver signal’s high level (level “1”). In a modern digital network, bit errors occur when a receiver fails to interpret the logic level of an incoming signal, predominantly due to the presence of noise. In most cases, extraneous noise that degrades the performance of a transceiver channel or communications system can be modeled as Gaussian noise as shown in Fig. 1. This characteristic allows the BER of such system to be determined using the Q factor [9].

![Gaussian noise distribution](image)

From Fig. 1, the optimum decision threshold, $y_d$, is the optimum setting where a $p_0(y_d) = p_1(y_d)$ condition results in a minimum BER. For the Gaussian noise model, the BER is given by (1).

$$BER = \frac{1}{2} \left[ \text{erfc} \left( \frac{\mu_1 - y_d}{\sqrt{2\sigma_1^2}} \right) + \text{erfc} \left( \frac{y_d - \mu_0}{\sqrt{2\sigma_0^2}} \right) \right]$$

Substituting the approximation $y_d = \frac{\sigma_0 \mu_1 + \sigma_1 \mu_0}{\sigma_0^2 + \sigma_1^2}$ in (1) gives (2).

$$Q = \sqrt{2-\text{erf}^{-1} \left( 1 - 2 \cdot BER \right)}$$

In the Q statistical method, the mathematical model derived from (2) and given in (3) approximates the Q factor [4].

$$Q = \sqrt{2-\text{erf}^{-1} \left( 1 - 2 \cdot BER \right)}$$

In this paper, the Q statistical method is applied on the CDR jitter tolerance measurements. CDR jitter tolerance is a measure of the amount of jitter than can be added to the incoming data before an error occurs. The Q factor can be obtained using (4) and (5) where random jitter is $RJ_{RMS}$, total deterministic jitter is $DJ_{P-P}$, and the minimum required eye opening at a specified BER is $T_{OPEN}$ as illustrated in Fig. 2 [10].

$$Q = \frac{T_{OPEN} - T_{OPEN} - DJ_{P-P}}{2 \times RJ_{RMS}}$$

$$RJ_{P-P} = 2Q_{BER} \times RJ_{RMS}$$

For most protocols, jitter tolerance is measured by a fixed amount of RJ and ISI where by PJ is swept to the maximum that the receiver can tolerate. By fixing RJ and ISI, the eye opening becoming narrower as PJ increases, as shown in Fig. 3. This figure also shows the linear decrease of Q with the increase in PJ, for a consistent number of bits transmitted. In short, the PJ has an inverse linear relationship with Q. This relationship can be extrapolated to obtain the PJ of lower BER (larger Q value) based on the PJ measurements from higher BER values.

![Relationship between CDR minimum eye opening and the Q-factor](image)

![Relationship between CDR minimum eye opening, Q-factor and PJ](image)

IV. METHOD

The following test case demonstrates the use of the Q statistical method to measure CDR jitter tolerance.

A. Measurement Setup
Fig. 4 shows the CDR jitter tolerance measurement setup on an Altera Stratix IV GX device’s transceiver block. An external reference clock is provided by the Agilent 81134A pulse generator. The NoiseCom noise generator is used to generate RJ while a backplane board is used to generate the ISI. An Agilent pattern generator produces a sine wave with PJ across different jitter frequencies using phase modulation. This signal is used as the clock signal for the ParBERT to generate jittery PRBS-31 data input to the receiver, passing through the CDR, and is routed back out through the transmitter via an internal loopback path. The amplitude of PJ is swept to the maximum that the receiver can tolerate for a range of frequencies. These measurements are compared against the test protocol’s sinusoidal jitter tolerance mask.

B. Data Extrapolation

This new method can be used to predict the PJ for a lower BER based on the PJ measurements from higher BER values. In this test case, we want to measure the CDR jitter tolerance for a BER of $10^{-12}$ at the data rates of 3.125 and 6.375 Gbps to determine compliance with the Interlaken protocol. However, a 95% confidence level for a BER of $10^{-12}$ will involve transmitting up to 3 trillion bits! This new method accelerates the characterization process by using the Q statistical method to predict the PJ values for the lower BER of $10^{-12}$ based on measured data for the higher BER values of $10^{-8}$ and $10^{-11}$.

Starting with a specific jitter frequency, the PJ at a lower BER of $10^{-12}$ is obtained. The PJ is then increased slightly to induce a higher BER. The relationship between PJ and BER is plotted after several increments as shown in Fig. 5.

The BER is converted to Q using (3). This is possible because of the linear relationship between Q and PJ as illustrated in Fig. 6. In this test case, the Q for the BER of $10^{-12}$ is 7. Using extrapolation, the PJ for BER of $10^{-12}$ can be estimated based on this linear relationship.

For a close estimation of Q, sufficient measurement points are necessary. As a rule of thumb, measurements are taken for BER at every decade near the BER being extrapolated, in this case $10^{-8}$, $10^{-9}$, $10^{-10}$, and $10^{-11}$. In most cases, measurements for these BER values involve only a small step increase in PJ (i.e. 5 ps).

This process is repeated for different jitter frequencies to obtain a complete set of jitter tolerance measurements.

C. Data Analysis

Fig. 7 compares the PJ measured data with the extrapolated BER values (extrapolated PJ data) for the Interlaken protocol with the data rates of 3.125 and 6.375 Gbps. On both data rates, the comparisons show that the extrapolated data match the BER measured data very closely.

In addition, Fig. 8 shows that the difference between the measurement and extrapolated data across jitter frequencies for the 6.375-Gbps and 3.125-Gbps data rates can be as low as 1.25 mUI and does not exceed 55 mUI.

These observations and results show that the extrapolated data correlate very well with the measured data. Therefore, this new method is able to estimate the BER values with good accuracy.
The main benefit of employing this method is the measurement time saved. As stated above, measurements are taken for BER at every decade near the BER being extrapolated. In the example above, 6 different BER points are taken. Even with these additional measurements, the savings are very significant.

At very low data rates, i.e. 155 Mbps, the time taken to obtain PJ amplitude at BER of $10^{-12}$ using the conventional method is 322.2 mins. By using this new method (plus 6 different BER points), the time taken to project the PJ amplitude based on higher BERs is 193 mins. The conventional method takes about 1.67 times longer to complete measurements. Similarly, the time taken to obtain PJ amplitude at BER of $10^{-15}$ (if the data rate is 6.375 Gbps) using the conventional method is 5.45 days whereas the new method only requires a total of 282.3 s. Here, the conventional method takes about 1668 times longer to complete measurements. Furthermore, Table 2 shows that the extrapolated data for BER of $10^{-15}$ still correlate well with the measured data with the difference of 26.31 mUI.

### D. Challenges Encountered

Some challenges and limitations need to be addressed when applying this new methodology. The proposed method using the Q-statistical method is most suitable when the error distribution is Gaussian. This method is effective when Q fully determines the BER, as in the case of jitter tolerance measurements. However, the methodology can still be employed if error distributions that are pattern-dependent or non-Gaussian can be modeled as a superposition of Gaussian distributions.

In order to accurately estimate jitter tolerance values for a lower BER by extrapolation, sufficient measurements should be obtained for higher BER values to adequately establish the linear relationship between Q and jitter tolerance values. The extrapolation accuracy increases with the number of points taken, but with the measurement time as the tradeoff. Hence the number of measurements required is a balance between measurement time and accuracy.

### V. CONCLUSIONS

In summary, the adoption of increasingly higher data rates in the industry is creating many challenges in terms of characterization quality and time. By applying the widely used Q statistic to jitter tolerance measurement, the new method proposed in this paper provides an efficient and accurate alternative to measure BER that significantly reduces measurement time. Furthermore, this methodology allows jitter tolerance values for lower BER values to be estimated, values that are impossible to obtain using conventional measurement methods.

By employing this new method, jitter tolerance characterization on the Altera Stratix IV GX device can be completed within 2 months compared to more than 4 months using the conventional method. This characterization was accomplished even with measurements for 24 protocols with 966 measurement points (including sub-protocols and across processes, voltages, temperatures, and frequencies) with data rates ranging from 155 Mbps to 8.5 Gbps, and support for BER of $10^{-12}$ and lower. These results lend confidence that this new method can be deployed by various industries to achieve fast and effective jitter tolerance characterization. Furthermore, it enables the design of circuits with improved performance by taking into account stringent customer requirements as the characterization of extremely low BER is now feasible on silicon within reasonable measurement period.

### ACKNOWLEDGMENT

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### REFERENCES