Quantum Computing: Concept and Realization

K. W. Kim, A. A. Kiselev, V. M. Lashkin, W. C. Holton, and V. Misra

North Carolina State University
Outline

- Classical vs. Quantum
- Performance vs. number of elements
- Gedanken quantum computer
- Implementations and comparison
- Our current proposal
  - Physics
  - Design
  - Future projections
Classical Bit vs. Quantum Qubit

Quantum Bit is any two-level quantum system, for example, Electron Spin

$1 = \begin{array}{c} \uparrow \\ \downarrow \end{array}$

$|1\rangle = \begin{array}{c} \uparrow \\ \downarrow \end{array}$

$0 = \begin{array}{c} \downarrow \\ \uparrow \end{array}$

$|0\rangle = \begin{array}{c} \downarrow \\ \uparrow \end{array}$

$\begin{array}{c} \uparrow \\ \downarrow \end{array} = ?$

It’s an error!

$\begin{array}{c} \uparrow \\ \downarrow \end{array} = C_1|1\rangle + C_0|0\rangle$

It’s a superposition!
Two spins:

Four states in superposition: $2^2$

$$= C_{00}|00\rangle + C_{01}|01\rangle + C_{10}|10\rangle + C_{11}|11\rangle$$

Entanglement of spins 1 and 2

N spins:

$2^N$ states in superposition

$$0\cdots 00 + 0\cdots 01 + \cdots + 1\cdots 11$$
Problem Solving: tractable vs. intractable

- A classical computer solves problems of type P
- An N-bit quantum computer solves exponential problems
Moore’s Law as a potential limitation

Moore's Law

Number of chip components ~ performance

Classical Age

Quantum Age

10^18
10^14
10^10
10^6
10^2

10^1
10^0
10^-1
10^-2
10^-3

Feature Size (microns)

CMOS

Quantum Device
Another side --- cryptography

Security enabled
by the Uncertainty Principle
and by the No-Cloning Theorem
Basic ideas of QC

- Information stored in spin 1/2 quantum systems (qubits)

- Quantum computation scheme

\[ |1\rangle = \begin{array}{c}
\text{spin up}
\end{array} \quad |0\rangle = \begin{array}{c}
\text{spin down}
\end{array} \]

Initialization

Processing

Measurement

\[ |\psi\rangle \rightarrow \hat{U}_M \ldots \hat{U}_2 \hat{U}_1 |\psi\rangle \rightarrow |\psi\rangle_f \]
Fundamental gates:
only two required for all operations

1. $\frac{\pi}{2}$ single bit rotation

2. Controlled NOT

\[ A \quad \text{XOR} \quad B \]
Quantum computer

* Hamiltonian (coupled)

\[ \hat{H} = \frac{1}{2} \omega_1 \hat{\sigma}_1 + \frac{1}{2} \omega_2 \hat{\sigma}_2 + \frac{1}{4} J \hat{\sigma}_1 \cdot \hat{\sigma}_2 \]
Information processing via coherent pulses

- use superpositions

\[
|00\rangle \xrightarrow{\frac{\pi}{2} (\omega_2 - J/2)} \frac{1}{\sqrt{2}} (|00\rangle + |01\rangle)
\]

- use entangled states

\[
|00\rangle \xrightarrow{\frac{\pi}{2} (\omega_1 + J/2)} \frac{1}{\sqrt{2}} (|00\rangle + |11\rangle)
\]

Parallel computation and speed up!
QC Requirements

- Possibility to address qubits individually
- Initialize qubits
- Perform one- and two-qubit operations
- Read-out final result
- Small decoherence rate compared to ops rate
- Scalability (the more qubits - the better)
Implementations

- Not solid state QC
  * Trapped ions
  * NMR on molecules
  * Electrons trapped on liquid He
- Solid state QC
  - Orbital degree of freedom
  - Spin degree of freedom
  - Macroscopic wavefunction in superconductor

NC STATE UNIVERSITY
Asymmetric III-V Quantum Dot Quantum Computer

Electric dipole qubit transitions with dipole-dipole coupling

Silicon pillar

Qubits unique by fabrication

Sanders, Kim and Holton
Phys. Rev. A 60 #5 4146 Nov 99
Electrons trapped in quantum dots coupled to terahertz cavity photons

III-V Quantum dots distinguishable electrically with each quantum dot containing one electron. Dot array in micro-cavity. Coupling via terahertz cavity modes.

Electric dipole qubit transitions

Sherwin, Imamoglu, and Montroy
Phys. Rev. A 60 #5 3508 Nov 99
Quantum dot spins and cavity QED

III-V quantum dot electron spins coupled through microcavity mode

Qubit coupling mediated by microcavity mode

Qubits individually addressed by tapered fiber tips

Imamoglu, Awschalom, Burkard, DiVincenzo et al
PRL 83 #20 4204 15 Nov 99
Coupled Nuclear Spins Arrayed in Silicon Quantum Computer

P – impurities with spin = ½ interact through trapped electron to form coupled system

SET used to measure spin-state of final state

Kane
Nature 393 133 14 May 1999
Electron Spin Transistor (Transpinor) for Quantum Computing

Qubits distinguishable by addressing. Coupling by exchange interaction.

in Silicon-Germanium Heterostructures

Wang et al
Quant-ph/9905096 11 June 1999

in Silicon Quantum Dots

Kim and Holton
<table>
<thead>
<tr>
<th>Author</th>
<th>Sanders, Kim and Holton</th>
<th>Sherwin, Imamoglu &amp; Montroy</th>
<th>Platzman &amp; Dykman</th>
</tr>
</thead>
<tbody>
<tr>
<td>Structure</td>
<td>Electrons trapped in III-V quantum dots</td>
<td>Electrons trapped in III-V quantum dots in cavity</td>
<td>Electrons on liquid He Surface</td>
</tr>
<tr>
<td>Storage Mechanism</td>
<td>Electronic states of trapped electrons</td>
<td>Electronic states of trapped electrons</td>
<td>Electronic states of trapped electrons</td>
</tr>
<tr>
<td>Qubit Distinguishability</td>
<td>Controlled size of quantum dots</td>
<td>Voltage pulse</td>
<td>Externally applied voltage</td>
</tr>
<tr>
<td>Single bit Ops Rate</td>
<td>$10^{13}$ Hz</td>
<td>$10^9$ Hz</td>
<td>$10^9$ Hz</td>
</tr>
<tr>
<td>Two bit Compute Rate</td>
<td>$10^{10}$ Hz</td>
<td>$10^8$ Hz</td>
<td>$10^7$ Hz</td>
</tr>
<tr>
<td>Decoherence Time</td>
<td>$10^{-6}$ s</td>
<td>$10^{-4}$ s</td>
<td>$10^{-4}$ s</td>
</tr>
<tr>
<td>Ops to Decoherence</td>
<td>$10^4$ ops</td>
<td>$10^4$ ops</td>
<td>$10^3$ ops</td>
</tr>
<tr>
<td>Initialization Process</td>
<td>77K Temperature</td>
<td>Low Temperature</td>
<td>0.1K Temperature</td>
</tr>
<tr>
<td>Readout Process</td>
<td>Optical emission from ensemble</td>
<td>TACIT photon detector within cavity</td>
<td>Electron extraction via tunneling</td>
</tr>
<tr>
<td>Scalability</td>
<td>50 qubits</td>
<td>&gt; 100 qubits</td>
<td>$10^9$ qubits</td>
</tr>
<tr>
<td>Author</td>
<td>Makhlin, Schon &amp; Shnirman</td>
<td>Buckard, Loss &amp; DiVincenzo</td>
<td>Imamoglu, Awschalom, Divincenzo et al</td>
</tr>
<tr>
<td>-------------------------------</td>
<td>---------------------------</td>
<td>-----------------------------</td>
<td>--------------------------------------</td>
</tr>
<tr>
<td>Structure</td>
<td>Cooper pair with superconducting box</td>
<td>Electrons trapped in III-V quantum dots</td>
<td>Electrons trapped in III-V Quantum Dots</td>
</tr>
<tr>
<td>Storage Mechanism</td>
<td>Josephson junctions</td>
<td>Magnetic states of trapped electrons</td>
<td>Magnetic states of trapped electrons</td>
</tr>
<tr>
<td>Qubit Distinguishability</td>
<td>Physical location</td>
<td>Magnetic field gradient</td>
<td>Physical location</td>
</tr>
<tr>
<td>Single bit Ops Rate</td>
<td>$10^{10}$ Hz</td>
<td>$10^{10}$ Hz</td>
<td>$10^{11}$ Hz</td>
</tr>
<tr>
<td>Two bit Compute Rate</td>
<td>$10^{11}$ Hz</td>
<td>$10^{10}$ Hz</td>
<td>$10^{10}$ Hz</td>
</tr>
<tr>
<td>Decoherence Time</td>
<td>$10^{-7}$ s</td>
<td>$10^{-9}$ s</td>
<td>$10^{-4}$ s</td>
</tr>
<tr>
<td>Ops to Decoherence</td>
<td>$10^4$ ops</td>
<td>10 ops</td>
<td>$10^6$ ops</td>
</tr>
<tr>
<td>Initialization Process</td>
<td>Low Temperature</td>
<td>77K Temperature</td>
<td>Not described</td>
</tr>
<tr>
<td>Readout Process</td>
<td>Coupling to normal state transistor</td>
<td>Photon Scattering</td>
<td>Interaction w. laser field &amp; photon emission</td>
</tr>
<tr>
<td>Scalability</td>
<td>20 qubits</td>
<td>Not discussed</td>
<td>&gt; 100 qubits</td>
</tr>
</tbody>
</table>
# Solid State Quantum Computers

## Parameter Comparison

### Spin Degree of Freedom

<table>
<thead>
<tr>
<th>Author</th>
<th>Kane</th>
<th>Vrijin, Yablonovitch, Wang et al.</th>
<th>Sanders, Kim and Holton</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Structure</strong></td>
<td>P-impurities in Si</td>
<td>Electrons trapped at P-impurities in Si/Ge</td>
<td>Electrons trapped in quantum dots in Si</td>
</tr>
<tr>
<td><strong>Storage Mechanism</strong></td>
<td>Nuclear magnetic states of P impurity</td>
<td>Magnetic states of trapped electrons</td>
<td>Magnetic states of trapped electrons</td>
</tr>
<tr>
<td><strong>Qubit Distinguishability</strong></td>
<td>Voltage applied locally to P-gate</td>
<td>Voltage applied locally to P-gate</td>
<td>Variable local magnetic field</td>
</tr>
<tr>
<td><strong>Single bit Ops Rate</strong></td>
<td>$10^4$ Hz</td>
<td>$10^{10}$ Hz</td>
<td>$10^{10}$ Hz</td>
</tr>
<tr>
<td><strong>Two bit Compute Rate</strong></td>
<td>$10^4$ Hz</td>
<td>$10^8$ Hz</td>
<td>$10^8$ Hz</td>
</tr>
<tr>
<td><strong>Decoherence Time</strong></td>
<td>$10^6$ s</td>
<td>$10^{-3}$ s</td>
<td>$10^{-3}$ s</td>
</tr>
<tr>
<td><strong>Ops to Decoherence</strong></td>
<td>$10^{10}$ ops</td>
<td>$10^5$ ops</td>
<td>$10^5$ ops</td>
</tr>
<tr>
<td><strong>Initialization Process</strong></td>
<td>0.8K Temperature</td>
<td>Low Temperature</td>
<td>Low Temperature</td>
</tr>
<tr>
<td><strong>Readout Process</strong></td>
<td>Charge transfer to singlet/triplet</td>
<td>Charge transfer to singlet/triplet</td>
<td>Charge transfer to singlet/triplet</td>
</tr>
<tr>
<td><strong>Scalability</strong></td>
<td>$10^6$ qubits</td>
<td>$10^6$ qubits</td>
<td>$10^6$ qubits</td>
</tr>
</tbody>
</table>
Electron Spins Trapped Beneath Coupled Quantum Dots

- Hamiltonian for a single quantum dot pair.

\[
H = \mu_B g B_1 S_1 + \mu_B g B_2 S_2 + J S_1 \cdot S_2
\]

- Exchange coupling between adjacent quantum dots.

\[
2J = \int_V u(\vec{r}_1 - \vec{r}_2)\psi_1(\vec{r}_1)\psi_1^*(\vec{r}_2)\psi_2(\vec{r}_2)\psi_2^*(\vec{r}_1)dV
\]
Typical Design Parameters

- Pillar radius ~50 nm
- Gate radius~15 nm
- Pillar height~100 nm
- SiO$_2$ region~15 nm
- Doping region~20 nm
- Donor concentration ~3.e18 cm$^{-3}$
- Temperature~1.6 K
Confining in the radial direction

- Strong electrostatic confinement in the radial and z-directions along with the SiO$_2$/Si interface potential barrier serves to confine a single electron in the quantum dot
Single electron occupancy

- Single electron occupancy in the quantum dot holds over a finite range of the gate voltage:
  - $0.23 < V < 0.31$ (Volts)
Exchange energy control

- A single electron trapped beneath each dot gate provides the magnetic spin utilized in the quantum computer. A gate intermediate to the gate that performs the electron trapping can serve to vary the coupling between a given pair of electrons.
Reconfigurable Quantum Computer Showing Transpinor Output Sensors

- Current conductors to generate time dependent magnetic field bias, enabling single qubit addressing
- Potential pads enabling single electron trapping
- Wave function distortion pads
- Charge transfer pad
- Transpinor output detectors on periphery

Address Array provides unique magnetic field at addressed qubit

- Pulsed current to generate local magnetic field
- Interconnect array X-axis
- Interconnect array Y-axis
- Pulsed Magnetic field into paper from blue & red currents
- Quantum Dot Electrode
- Pulsed Magnetic field out of paper from blue & red currents
- Addressed qubit
Qubit Addressing

- For an external magnetic field $= 3.0$ Tesla
- The resonant microwave frequency $= \omega = 94$ GHz
- And with a line width $\sim 0.3$ gauss or 1 MHz
- And requiring a magnetic field address on 30x line width $= 9$ gauss
- A current in the addressing wire $= 1.12 \times 10^{-4}$ amp
- And with a wire dimension of 100x100 angstrom
- The current density $= 5 \times 10^7$ amp/cm²

- This is just at the threshold for electro migration for dc current at RT, but is OK for our application of pulses at low temperature.
Pulsed Microwave Field Generated Using a Microstrip Resonator

**Diagram Description:**
- **Permanent Magnet** provides a static magnetic field.
- **Quantum Dot Quantum Computer Silicon Chip** interacts with strong microwave magnetic fields.
- **Pin diode bias** allows for rapid turning on and off of the H field.
- **Ceramic Substrate** supports the chip and interconnect wires.
- **Interconnect wires** on the chip generate a pulsed magnetic field at each qubit.
- **Critical coupling** ensures efficient energy transfer.

**Additional Notes:**
- Pin diode bias for rapidly turning on and off the H field.
- Interconnect wires on chip to generate pulsed magnetic field at each qubit. Chip pin-out not shown in this figure.
Fabrication of Silicon Q-Dot Array
Q-Computer

- Layout and addressing for dynamic Quantum Computer control analogous to DRAM design and manufacture. (number of metal/dielectric layers considerably less than for 1Mbit DRAM).
- Resulting Quantum Computer chip coupled to microwave radiation field at ~ 94 GHz by placement in stripline cavity.
  - **External magnetic field ~ 3.0 Tesla provided by permanent magnet outside the microwave cavity.**
- Readout achieved by charge transfer via SETs on periphery (not shown) dependent on spin orientation (similar to readout for other proposed Quantum Computer designs based on spin).
Distinct Advantage of Design

- Scalable using mainstream silicon technology.
  - 1,000,000 qubits.
- Hi-speed single bit ops rate and compute rate.
  - Both readily tunable.
- Randomly and individually addressable qubits.
- Large number of ops before loss of coherence.
  - 100,000 ops to coherence loss.
- Dynamically Reconfigurable.